QCD Factorization and Evolution for SSA

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Based on work with Collins, Ji, Kang, Kouvaris, Sterman, Vogelsang, Yuan, …

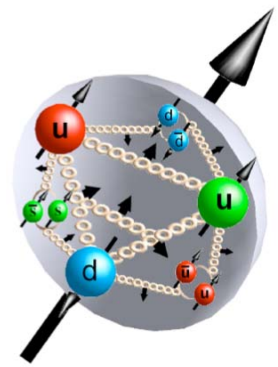
Spin of a composite particle

❑ Spin of a nucleus:

- ❖ Nuclear binding: 8 MeV/nucleon << mass of nucleon
- ❖ Nucleon number is fixed inside a given nucleus
- ❖ Spin of a nucleus = sum of the valence nucleon spin

❑ Spin of a nucleon – Naïve Quark Model:

- ❖ If the probing energy << mass of constituent quark
- ❖ Nucleon is made of three constituent (valence) quark
- ❖ Spin of a nucleon = sum of the constituent quark spin



❑ Spin of a nucleon – QCD:

- ❖ Current quark mass << energy exchange of the collision
- ❖ Number of quarks and gluons depends on the probing energy

Proton spin in QCD

❑ **Angular momentum of a proton at rest:**

$$S = \sum_f \langle P, S_z = 1/2 | \hat{J}_f^z | P, S_z = 1/2 \rangle = \frac{1}{2}$$

❑ **QCD Angular momentum operator:**

Energy-momentum tensor

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x\ M^{0jk}_{\rm QCD} \quad \Longleftarrow \quad M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\, x^\mu - T^{\alpha\mu}_{\rm QCD}\, x^\nu$$

Angular momentum density

❖ Quark angular momentum operator:

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma}\gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D}))\, \psi_q\right]$$

❖ Gluon angular momentum operator:

$$\vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B})\right]$$

Need matrix elements of these partonic operators

Sum rule for proton spin

- **Partonic contribution to the proton spin:**

If $\vec{P}=0$, $\langle P,S|\vec{J}_{q,g}(\mu^2)|P,S\rangle \propto \vec{S}$

$$\Longrightarrow \langle P,S|\vec{J}_{q,g}(\mu^2)|P,S\rangle \equiv J_{q,g}(\mu^2)\,2\vec{S}$$

Quark contribution: $J_q(\mu^2)$ Gluon contribution: $J_g(\mu^2)$

- **Ji's sum rule:**

$$\frac{1}{2} = J_q(\mu^2) + J_g(\mu^2) = \left[\frac{1}{2}\Sigma(\mu^2) + L_q(\mu^2)\right] + J_g(\mu^2)$$

Quark helicity: $\Sigma(\mu^2) = \int_0^1 dx \sum_f \left[\Delta q_f(x,\mu^2) + \Delta\bar{q}_f(x,\mu^2)\right]$

- **Calculation of these matrix elements:**

  - Proton wave function in terms of quarks and gluons – unknown
  - Lattice QCD: non-local operators

Foundation of perturbative QCD

- ❑ Renormalization
  – QCD is renormalizable

Nobel Prize, 1999
't Hooft, Veltman

- ❑ Asymptotic freedom
  – weaker interaction at a shorter distance

Nobel Prize, 2004
Gross, Politzer, Welczek

- ❑ Infrared safety
  – pQCD factorization and calculable short distance dynamics

Question

- Can we measure hadronic matrix elements of simple quark or gluon operators?

  Experiments measure hadronic cross sections

  Many parton could participate in the hadronic collisions

- Approximation:

  High energy scattering is dominated by single parton collision

- Factorization – large momentum transfer:

  Single identified hadron – inclusive DIS – structure functions

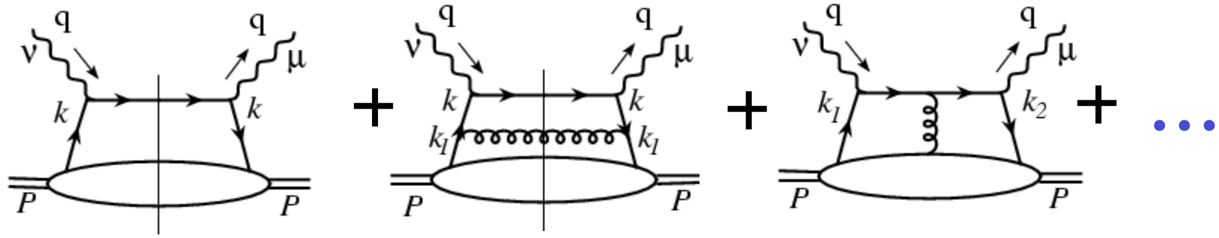
  Two identified hadrons – Drell-Yan, SIDIS, …

  Processes with more than two identified hadrons
  – hadronic pion production, …

Inclusive DIS – one identified hadron

- Feynman diagram representation of the DIS scattering:

$$W^{\mu\nu} \propto$$



- Perturbative pinched poles:

$$\int d^4k\ \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) \Rightarrow \infty \text{ perturbatively}$$

Dominated by a region where $k^2 \sim M^2 << Q^2$ – "long-lived" parton state

- Perturbative factorization:

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

Nonperturbative matrix element

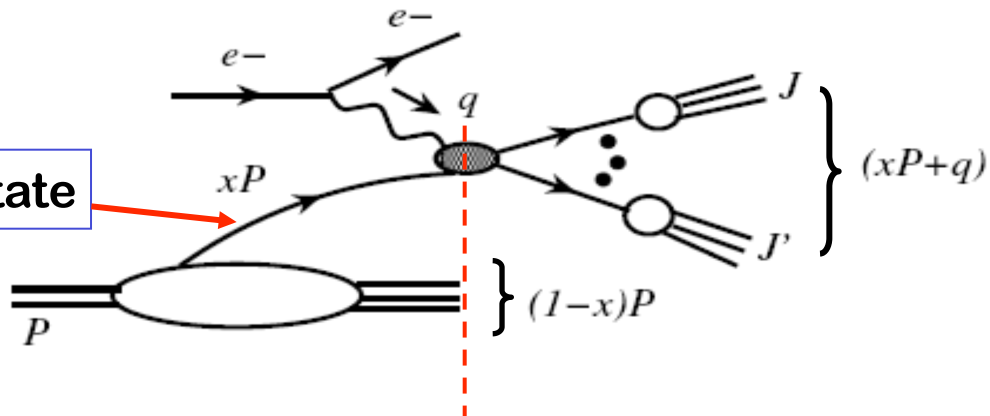
$$\int \frac{dx}{x} d^2k_T\ \mathrm{H}(Q,k^2=0)\ \int dk^2 \left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0})$$

Short-distance

Picture of DIS factorization

- **Time evolution:**

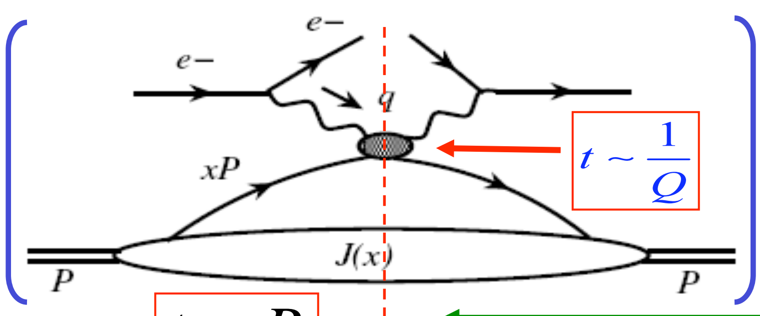
Long-lived parton state



Time: "Past" Now "Future"

- **Unitarity – summing over all hard jets:**

$$\sigma_{\text{tot}}^{\text{DIS}} \propto \text{Im}$$



$t \sim \frac{1}{Q}$

$t \sim R$

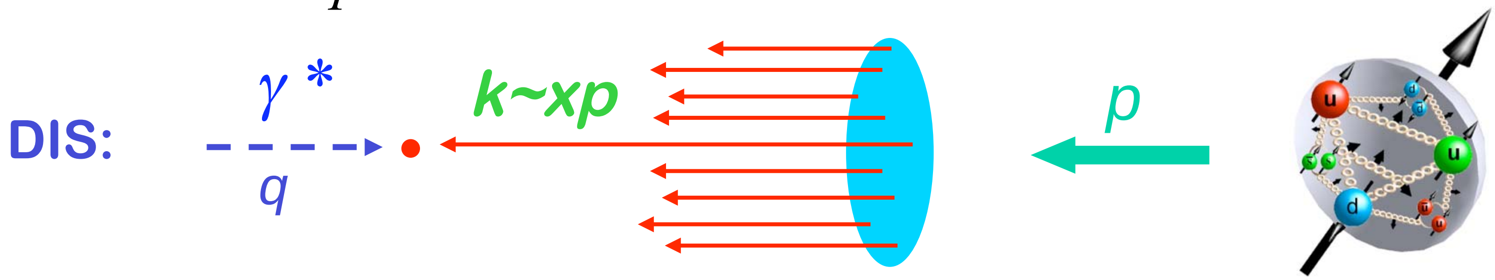
Not IR safe

Interaction between the "past" and "now" are suppressed!

Collinear Factorization

❑ Collinear approximation:

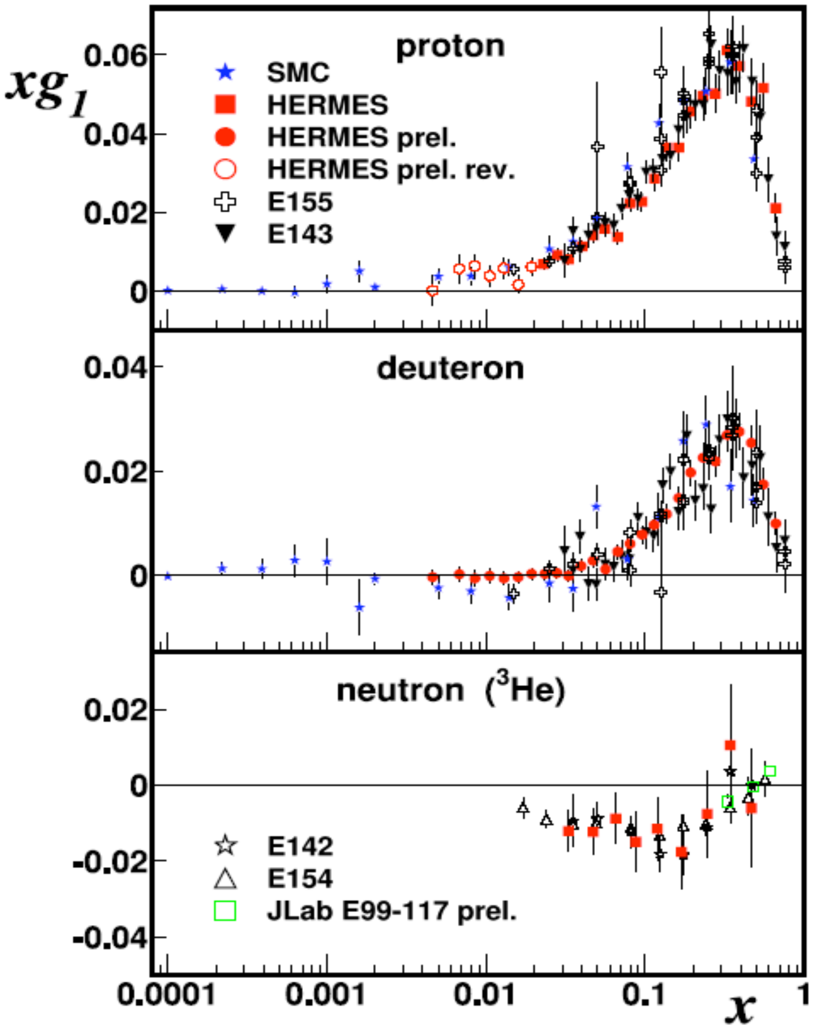
$$k^{\mu} \approx xp^{\mu} + \frac{k_T^2}{2xp \cdot n} n^{\mu} + k_T^{\mu} \approx xp \qquad \text{if} \quad Q \sim xp \cdot n \gg k_T, \sqrt{k^2}$$

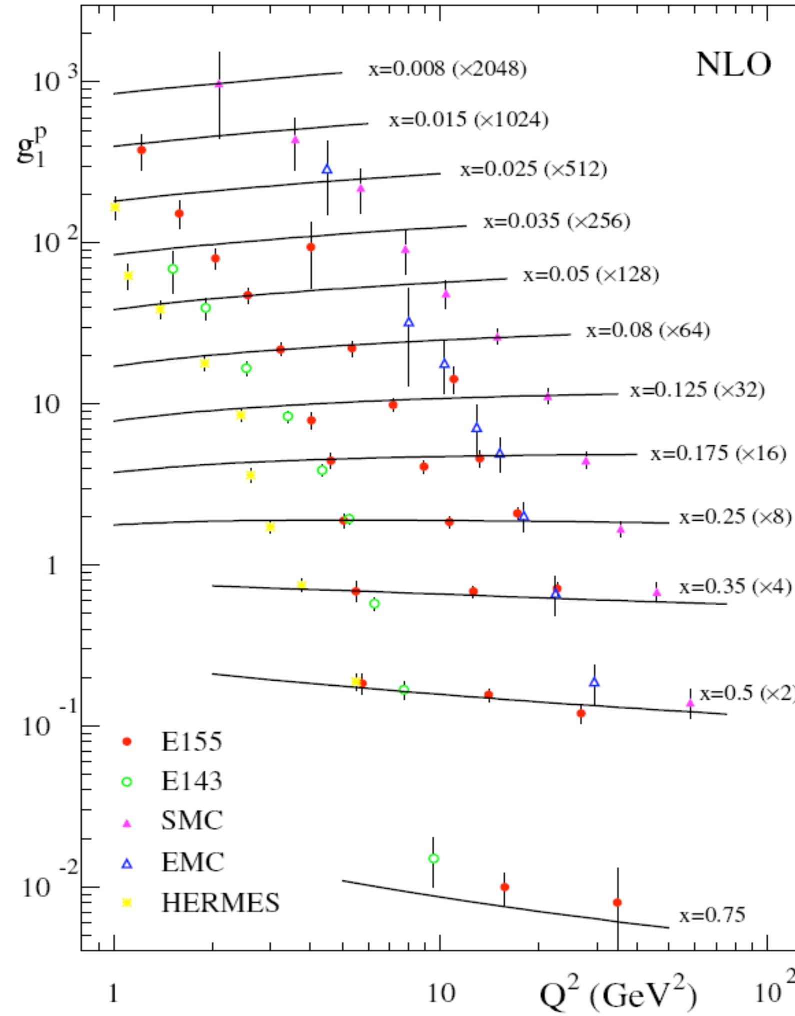


- ❖ Hadron is approximated by a beam of partons of momentum fraction x_i
- ❖ Parton's transverse motion is integrated into parton distributions: $\varphi(x)$

❑ Parton distributions are process independent, and QCD collinear factorization has been very successful

Polarized inclusive DIS

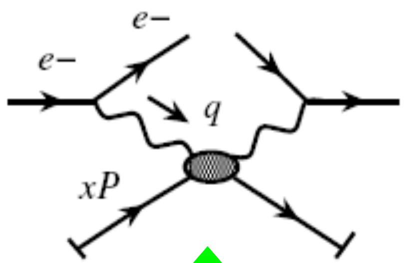


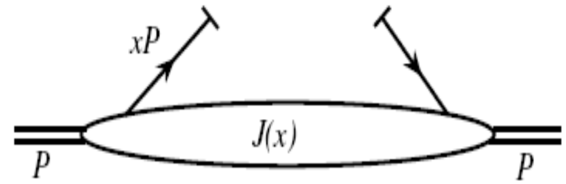


NLO QCD factorization is consistent with the data

Factorization – two identified hadrons

- One hadron:

$\sigma_{\text{tot}}^{\text{DIS}} \sim$ 

$\otimes$ 

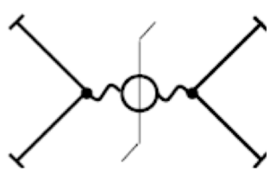
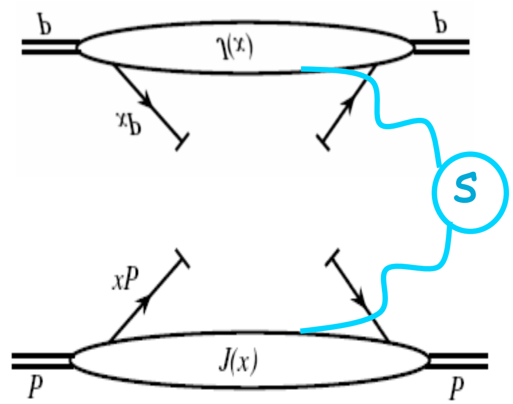
$+\ O\left(\dfrac{1}{QR}\right)$

Now
Hard-part

Past
Parton-distribution

Connection
Power corrections

- Two hadrons:

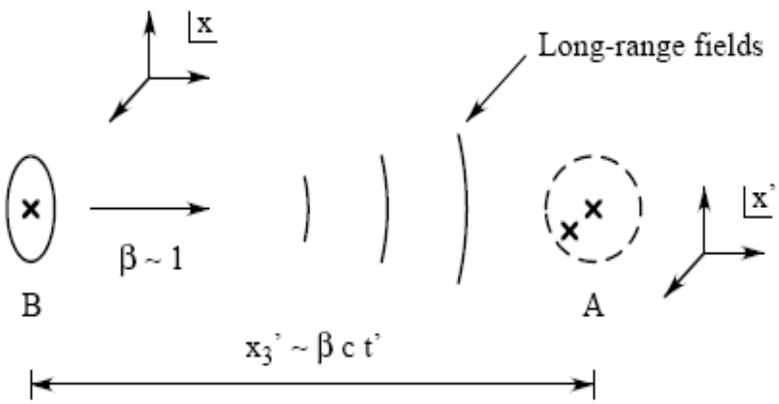
$\sigma_{\text{tot}}^{\text{DY}} \sim$ 
$\otimes$ 

$+\ O\left(\dfrac{1}{QR}\right)$

Soft interactions between incoming hadrons break the universality of PDFs

Heuristic argument for factorization

- Soft-gluon interactions take place all the time:
- Suppression of soft-gluon interactions:



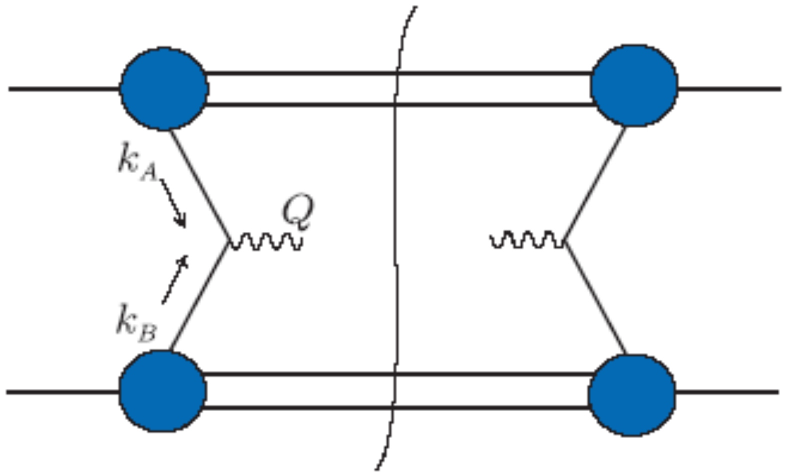
x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2 + \gamma^2\Delta^2)^{1/2}}$
	$\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2 + \gamma^2\Delta^2)^{3/2}}$
	$\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

- Factorization breaks beyond $1/Q^2$ ($1/Q$ for spin):

$$\sigma(Q) = H^0 \otimes f_2 \otimes f_2 + \left(\frac{1}{Q^2}\right) H^1 \otimes f_2 \otimes f_4 + O\left(\frac{1}{Q^4}\right)$$

Doria, et al (1980)
Basu et al. (1984)
Brandt, et al (1989)

Why Drell-Yan factorization makes sense?



- Pinch singularities
- Long-lived partonic states
- lowest order kinematics determines the process

$$\frac{d\sigma}{dQ^2 dy} = \int dk_{A,T}\, dk_{B,T} dk_A^- dk_B^+ \; H_{\mu,\nu}(Q^+, Q^-, k_{A,T} + k_{B,T})$$
$$\times \mathrm{Tr}\{\gamma^\mu \Phi_A(Q^+ - \cancel{k_B^+}, k_{A,T}, k_A^-)\gamma^\nu \Phi_B(k_B^+, k_{A,T}, Q^- - \cancel{k_A^-})\}$$

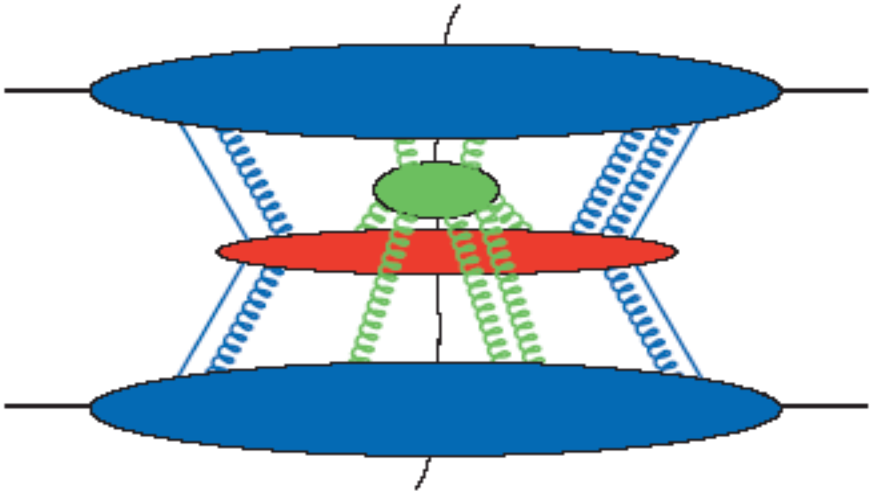
Approximation:

$$k_{A,T}^2 \;,\; k_{B,T}^2 \ll Q^2$$
$$k_A^- \ll Q^-$$
$$k_B^+ \ll Q^+$$

$\Longrightarrow$ **Drell-Yan formula**

QCD dynamics is rich and complicate

❑ Leading pinch surface:



Analysis of leading (pinch or singular) integration regions gives the following:

Hard (Large P_T or way off shell)

Collinear (to A or to B, small P_T) – one-pair "physical parton" from each hadron

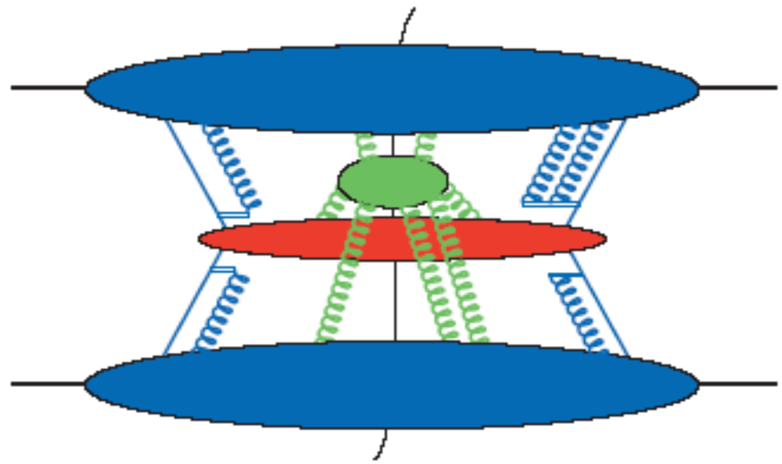
Soft (All components small, includes "Glauber.")

❑ Factorization:

Long-distance distributions are process independent

Eikonalization of collinear gluons

The extra collinear gluons would be a big problem because the factorization formula contemplates collisions of only one parton from each hadron.



But the collinear gluons are OK

- The extra collinear gluons have $\epsilon^\mu \propto k^\mu$.

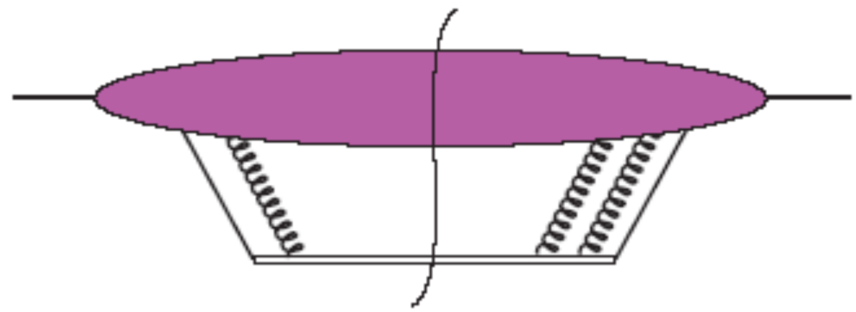
- There effect can be approximated as shown with eikonal lines, with u in the $-$ direction for hadron A, u in the $+$ direction for hadron B,

$$\text{propagator} = \frac{i}{k \cdot u + i\epsilon}$$

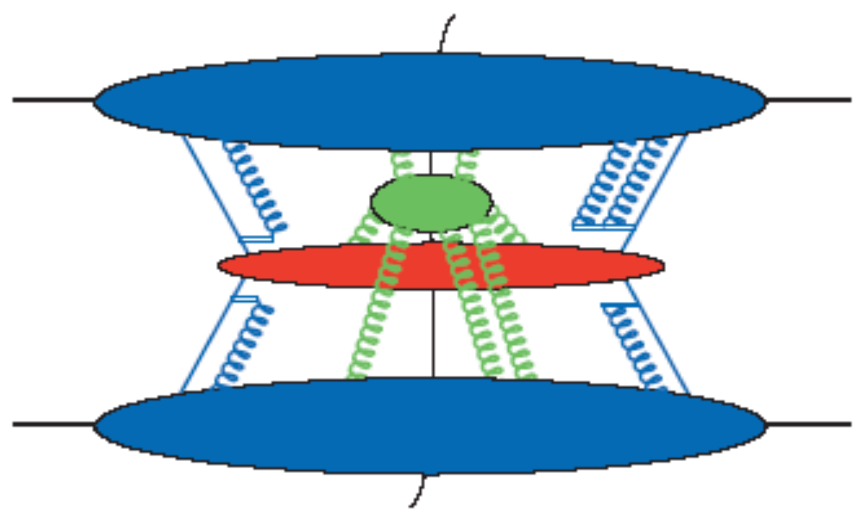
$$\text{vertex} = -igt_a u^\mu$$

Factorization of PDFs

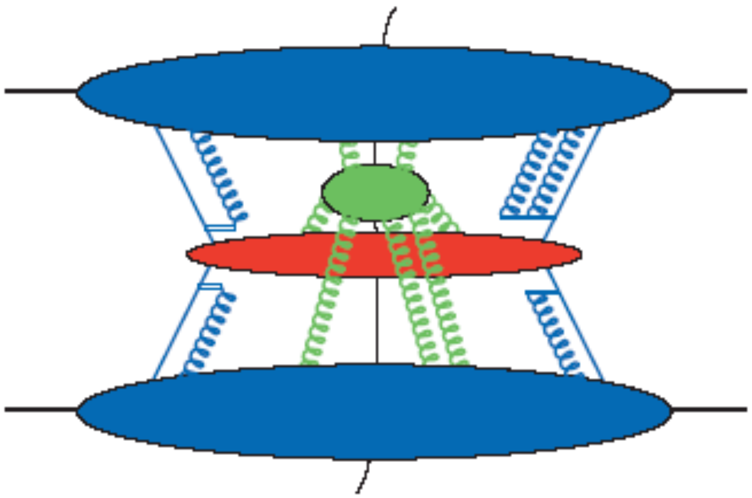
Parton distribution in diagrams

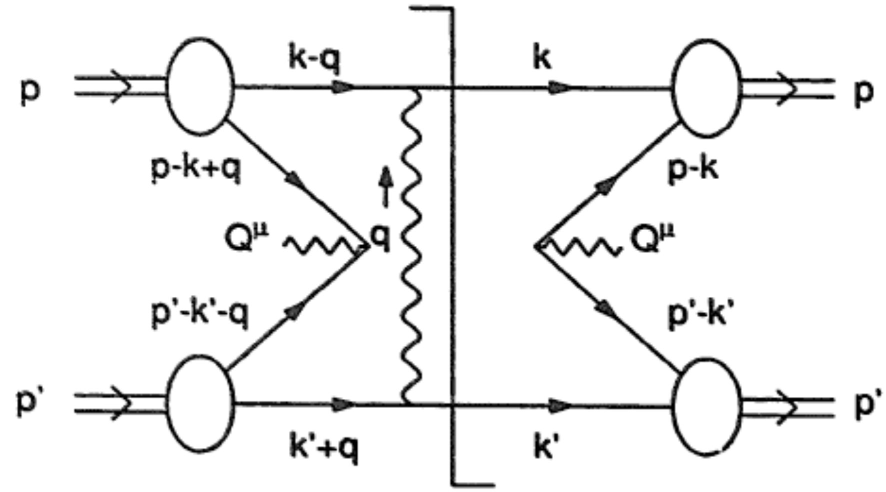


Compare



Trouble from soft gluons



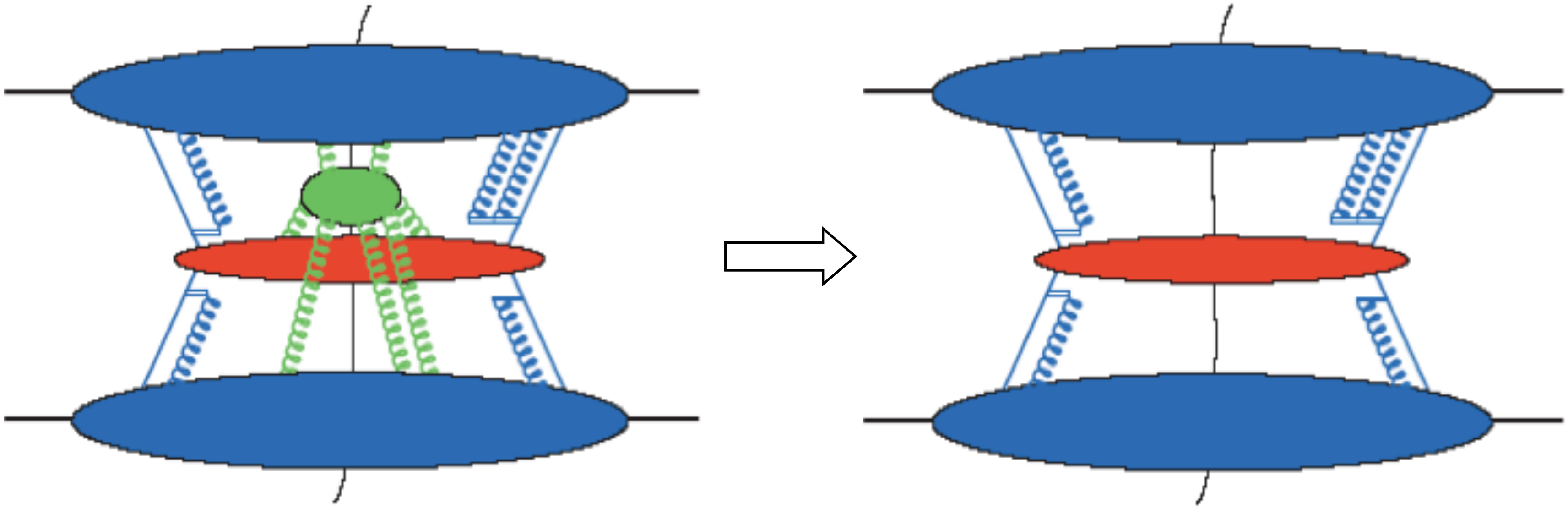


- It seems that a soft gluon exchanged from a spectator quark in hadron A to the active quark in hadron B can rotate the quark's color and thus keep it from annihilating.

- Soft gluon approximations (with eikonal lines) needs $q^\pm$ not too small. But $q^\pm$ contours can be trapped in "too small" region.

Pinch from spectator interaction: $q^\pm \sim M^2/Q \ll q_\perp \sim M$

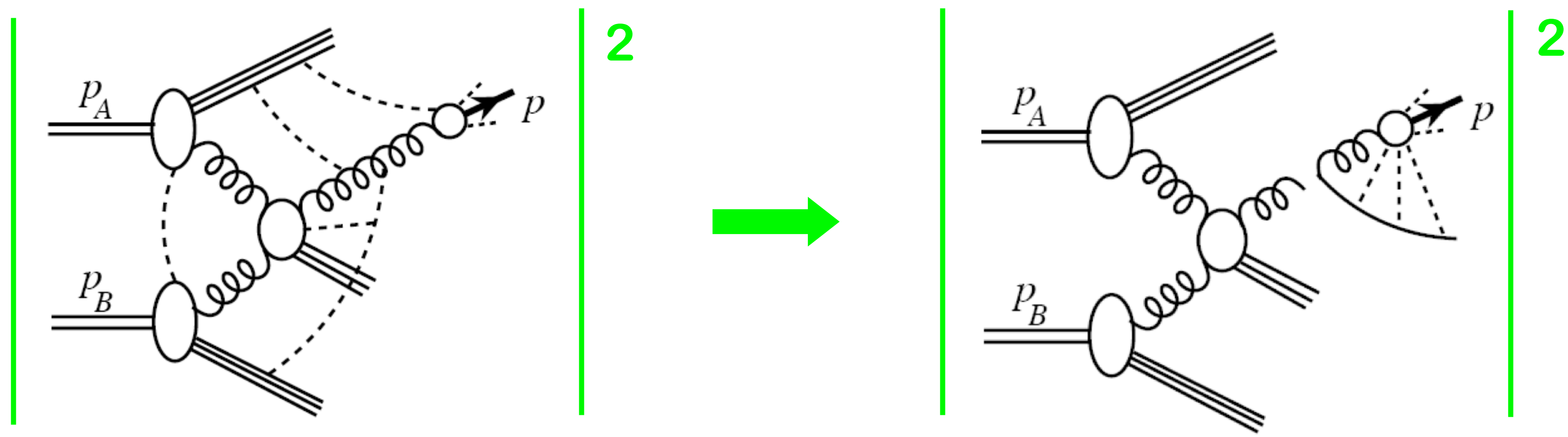
Soft gluons take care of themselves



- ❖ **Most technical part of the factorization**
- ❖ **Sum over all final states to remove all poles in one-half plane**
  – no more pinch poles
- ❖ **Deform the $q^{\pm}$ integration out of the trapped soft region**
- ❖ **Eikonal approximation, unitarity, causality, and gauge invariance**

Factorization – high P_T single particle

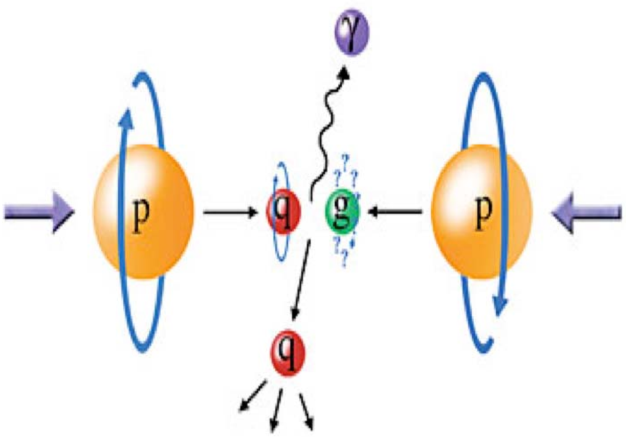
Nayak, Qiu, Sterman, 2006



- ❖ Eikonalization of gluons collinear to the final-state hadron
- ❖ Factorization of the fragmentation function
- ❖ Factorization of gluons from the initial-state hadrons
  – same as the factorization of Drell-Yan
- ❖ Normalization of short-distance hard parts is fixed by the definition of the universal PDFs and FFs

Factorization – approximation

❑ **Hadronic production of direct photon:**



$$\frac{d\sigma}{dy dp_T^2} = \int \frac{dx}{x} q(x) \int \frac{dx'}{x'} g(x') \frac{d\hat{\sigma}_{qg\to\gamma q}}{dy dp_T^2}$$
$$+ \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{1}{p_T^\alpha}\right)$$

❑ **Predictive power:**

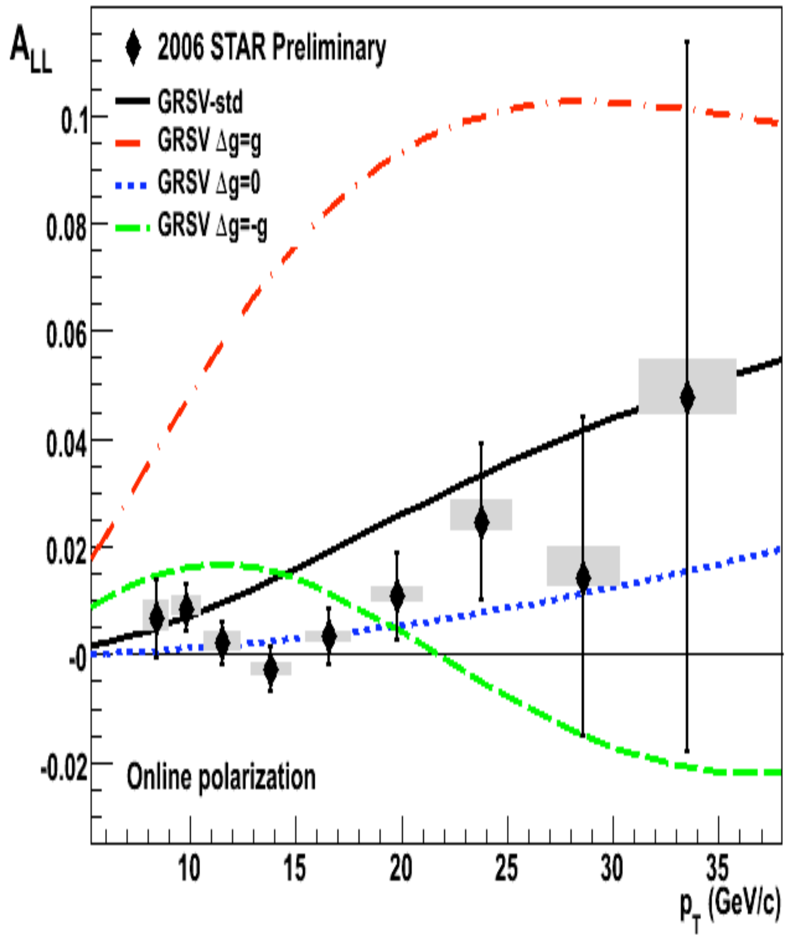
❖ short-distance and long-distance physics are separately gauge invariant

❖ short-distance part is infrared-Safe, and calculable

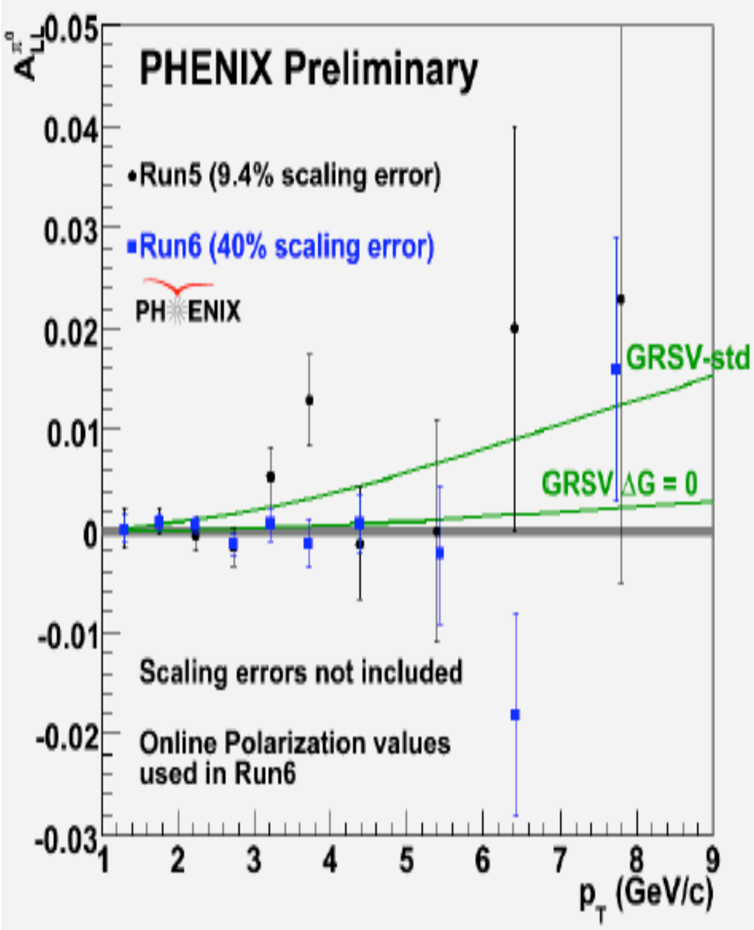
❖ long-distance part is process independent – Universal PDFs

Polarized hadronic collisions

Star jet



Phenix π^0



Small asymmetry leads to small gluon "helicity" distribution

Quark "helicity" to proton spin

- **Extracted by the leading power QCD:**

$$\Delta q = \int_0^1 dx\, \Delta q(x) = \langle P, s_{||} | \overline{\psi}_q(0) \gamma^+ \gamma_5\, \psi_q(0) | P, s_{||} \rangle$$

  - Integrated over "ALL" momentum components of active parton
  - Collinear factorization:
    - parton entering the hard part has only collinear momentum
    - parton in the distribution has all components

- **NLO QCD global fit - DSSV:**

$$\Delta u + \Delta \bar{u} = 0.813 \qquad \Delta d + \Delta \bar{d} = -0.458 \qquad \Delta \bar{s} = -0.057$$

$$\Sigma = 0.242 \approx 24\% \text{ proton spin}$$

de Florian, Sassot, Stratmann, and Vogelsang
Phys. Rev. Lett. 2008

- **From Ji's definition:**

$$J_q = \frac{1}{2} \int d^3x\, \langle \vec{P} = 0, \vec{S} |\, \psi_q^\dagger(x) \vec{\gamma} \cdot \vec{S} \gamma_5\, \psi_q(x)\, | \vec{P} = 0, \vec{S} \rangle + L_q$$

Gluon "helicity" to proton spin

❑ Extracted by the leading power QCD:

$$\Delta g = \int_0^1 dx\,\Delta g(x) = \langle P, s_\parallel | F^{+\mu}(0) F^{+\nu}(0) | P, s_\parallel \rangle (-i\epsilon_{\mu\nu})$$

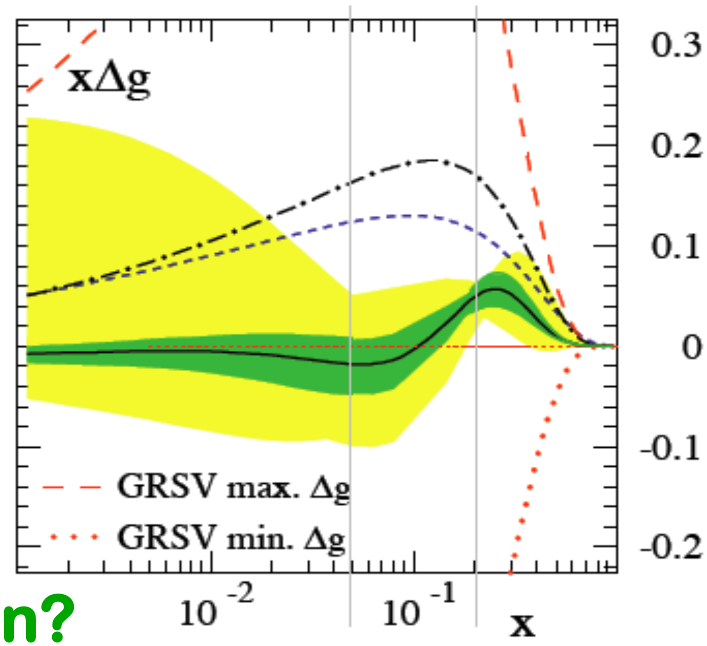
Integrated over "ALL" momentum components of active gluon

❑ NLO QCD global fit - DSSV:

$$\Delta g = -0.084$$

arXiv:0804.0422

- ❖ $\Delta g(x)$ change sign in RHIC region
- ❖ Effectively, "no" contribution to proton spin



❑ Measure the gluonic contribution?

$$J_g = \frac{1}{2} \int d^3x \, \langle \vec{P} = 0, \vec{S} | \, [\vec{x} \times \vec{E}(x) \times \vec{B}(x)] \cdot \vec{S} \, | \vec{P} = 0, \vec{S} \rangle$$

Questions

How to go beyond
the probability distributions?

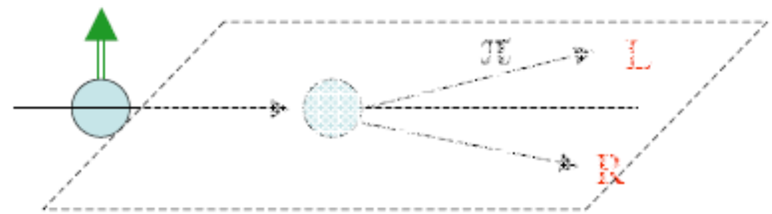
How to probe
parton's transverse motion?

Single Transverse-Spin Asymmetry (SSA)

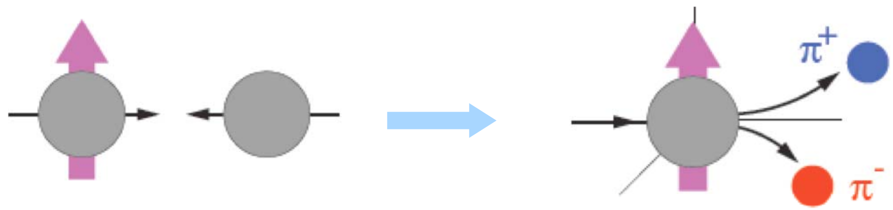
$$A(\ell,\vec{s}) \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

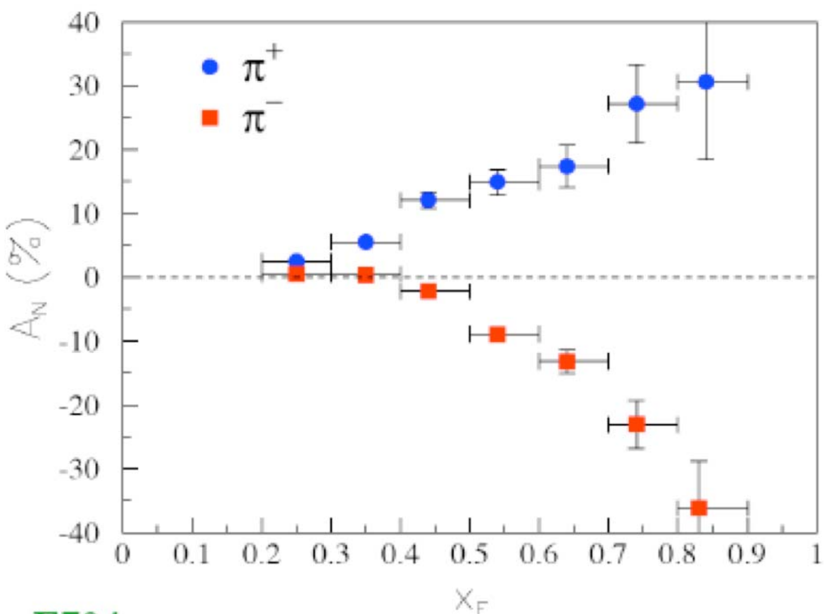
SSA in hadronic collisions

- **Hadronic** $p\uparrow + p \rightarrow \pi(l)X$ **:**



$$A_N = \frac{1}{P_{\rm beam}} \frac{N^{\pi}_{\rm left} - N^{\pi}_{\rm right}}{N^{\pi}_{\rm left} + N^{\pi}_{\rm right}}$$





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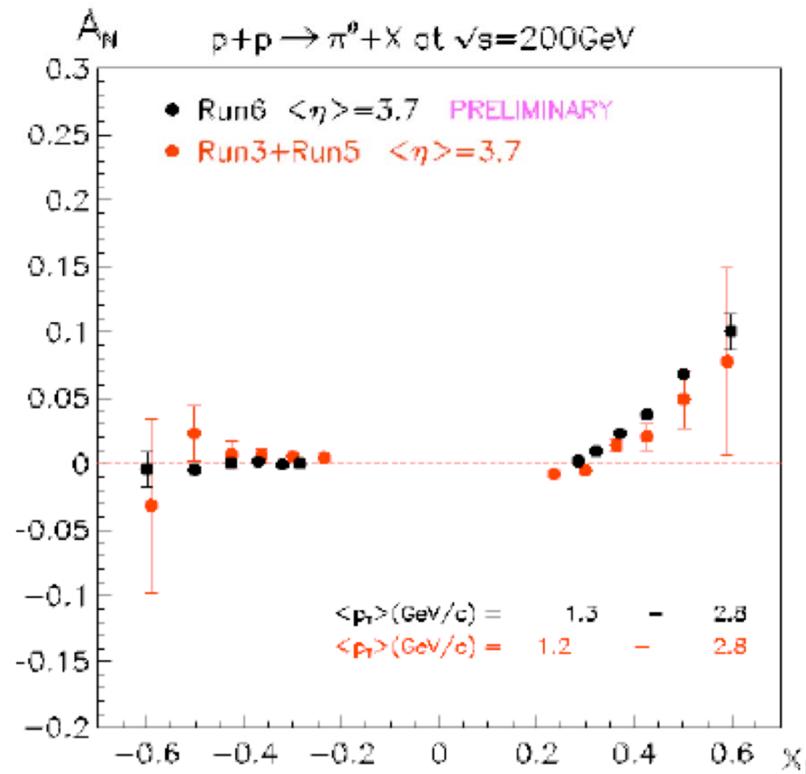


STAR (BRAHMS, too)

Role of fundamental symmetries

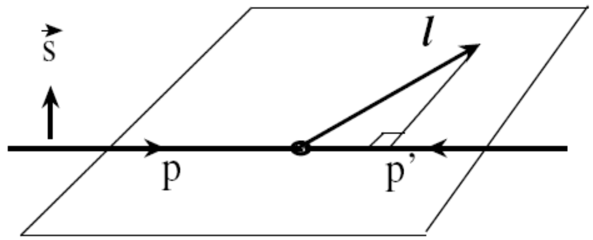
- **Fundamental symmetry and vanishing asymmetry:**
  - A_L=0 (longitudinal) for Parity conserved interactions
  - A_N =0 (transverse) for inclusive DIS – Time-reversal invariance

    – proposed to test T-invariance by Christ and Lee (1966)

  Even though the cross section is finite!

- **SSA corresponds to a T-odd triple product**

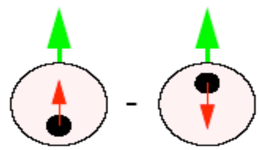
$$A_N \propto i\vec{s}_p \cdot \left(\vec{p} \times \vec{\ell}\right) \Rightarrow i\varepsilon^{\mu\nu\alpha\beta} p_\mu s_\nu \ell_\alpha p'_\beta$$



→ Novanishing A_N requires a phase, a spin flip, and enough vectors to fix a scattering plan

SSA in parton model

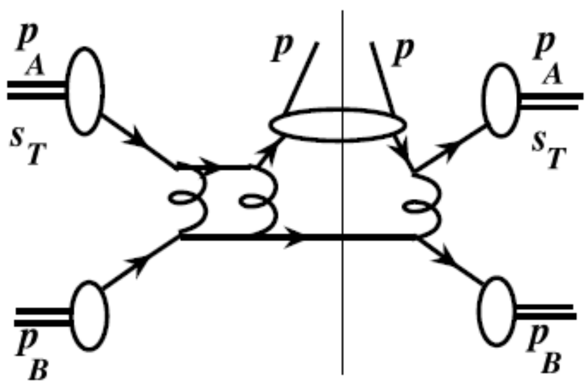
❑ **The spin flip at leading twist – transversity:**

$\delta q(x) =$  $\propto \langle P, \vec{S}_\perp | \overline{\psi}_q \left[\gamma^+ \gamma \cdot \vec{S}_\perp \right] \psi_q | P, \vec{S}_\perp \rangle$

Chiral-odd helicity-flip density

❖ the operator for δq has even γ's ⟹ quark mass term

❖ the phase requires an imaginary part ⟹ loop diagram



$\sim m_q \rightarrow 0$

⟹ **SSA vanishes in the parton model**
connects to parton's transverse motion

Cross section with ONE large scale

- Collinear factorization approach is more relevant

$$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$$ – Expansion

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

Too large to compete!

Three-parton correlation

- SSA – difference of two cross sections with spin flip is power suppressed compared to the cross section

$$\Delta\sigma(Q, s_T) \equiv [\sigma(Q, s_T) - \sigma(Q, -s_T)]/2$$

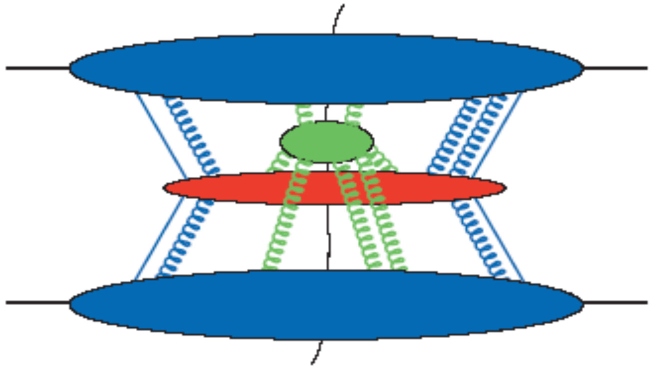
$$= (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F) + \mathcal{O}(1/Q^2)$$

- Sensitive to twist-3 mu;ti-parton correlation functions
- Integrated information on parton's transverse motion

Factorization beyond leading power – I

Qiu, Sterman, 1991

❑ Power correction from the leading pinch surface:



❖ For "central soft" gluons (similar to Glauber gluons):
Expand both jet functions around the leading twist one

$$\Delta J^{\{\mu_i\}(C)}(p,k,q_i,C_S) = J^{\{\mu_i\}(C)}(p,k,q_i,C_S) - J^{\{+\}(C)}(p,k,\tilde{q}_i,C_S)$$

$$\Rightarrow\quad J^{\{\mu_i\}(C)}(p,k,q_i,C_S)\, S^{(C_S)}_{\{\mu_i\}\{\nu_j\}}(q_i,q_j')\, J'^{\{\nu_j\}(C')}(p',k',q_j',C_S)\, H(k,k',q_i,q_j',C_S)$$

$$= \Big[\Delta J^{\{\mu_i\}(C)}(p,k,q_i,C_S)\, S^{(C_S)}_{\{\mu_i\}\{\nu_j\}}(q_i,q_j')\, \Delta J'^{\{\nu_j\}(C')}(p',k',q_j',C_S) \quad \longleftarrow \mathcal{O}(M^4/Q^4)$$

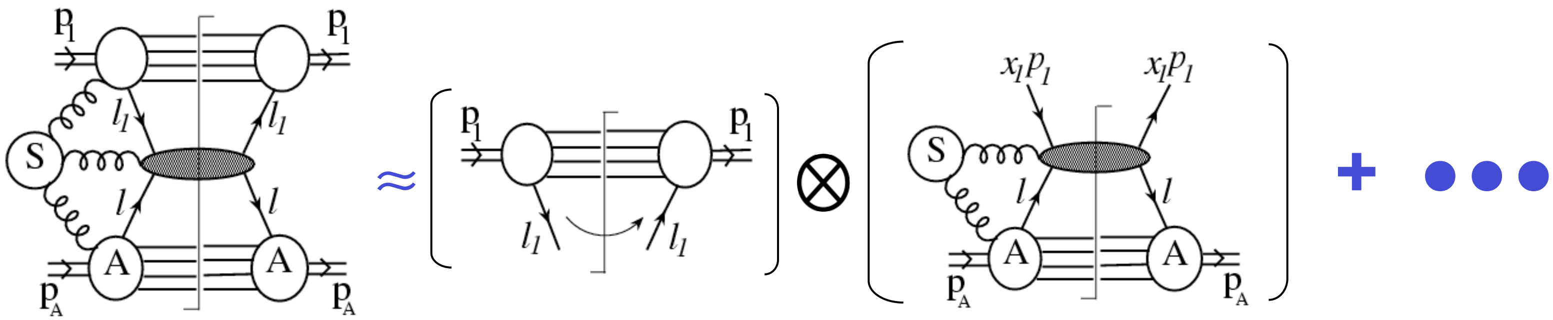
$$+ J^{\{+\}(C)}(p,k,\tilde{q}_i,C_S)\, S^{(C_S)}_{\{+\}\{\nu_j\}}(q_i,q_j')\, \Delta J'^{\{\nu_j\}(C')}(p',k',q_j',C_S) \quad \longleftarrow \mathcal{O}(M^2/Q^2)$$

$$+ \Delta J^{\{\mu_i\}(C)}(p,k,q_i,C_S)\, S^{(C_S)}_{\{\mu_i\}\{-\}}(q_i,q_j')\, J'^{\{-\}(C')}(p',k',\tilde{q}_j',C_S) \quad \longleftarrow \mathcal{O}(M^2/Q^2)$$

$$\text{LP} \longrightarrow\quad + J^{\{+\}(C)}(p,k,\tilde{q}_i,C_S)\, S^{(C_S)}_{\{+\}\{-\}}(q_i,q_j')\, J'^{\{-\}(C')}(p',k',\tilde{q}_j',C_S)\Big]\, H(k,k',q_i,q_j',C_S)$$

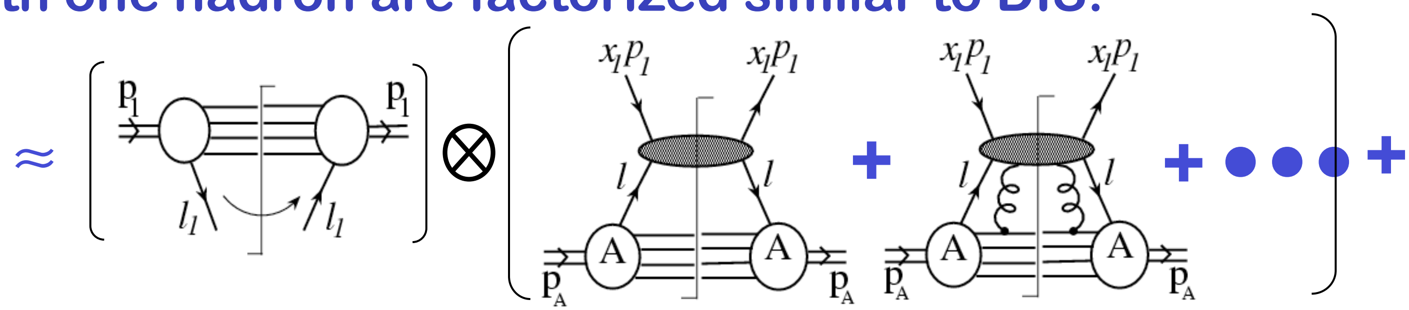
Factorization beyond leading power – II

❖ Standard factorization of twist-2 PDFs:



No communication between hadrons other than the active partons

❖ Diagrams with one hadron are factorized similar to DIS:



Soft-gluon interaction between the sub-leading contribution of two hadrons cannot be factorized!

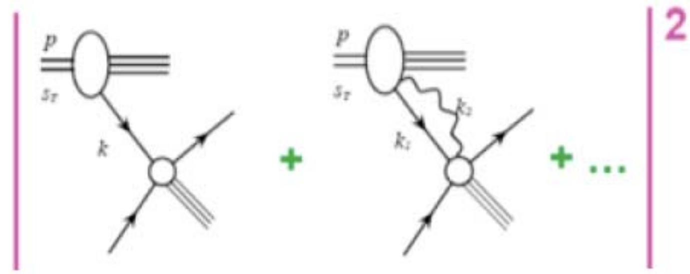
3-parton for SSA

Factorization with two or more hadrons only works upto the first subleading power!

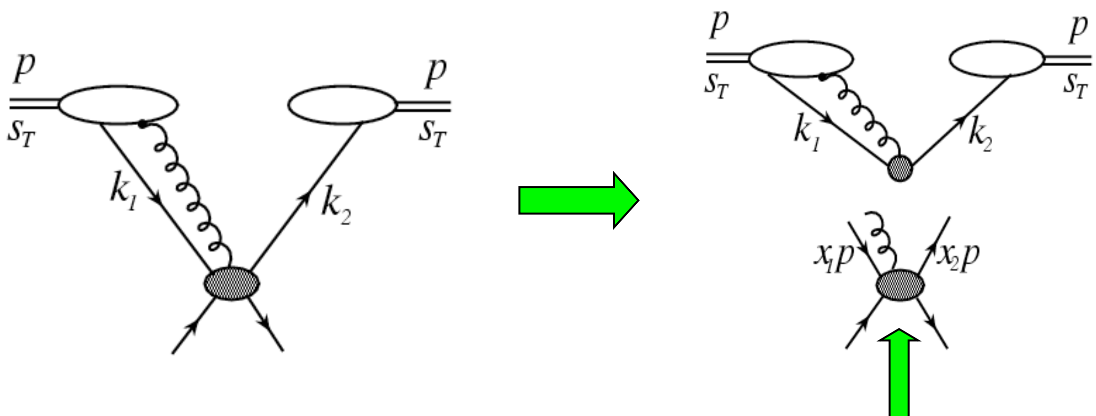
SSA in QCD Collinear Factorization – I

- **All scales >> Λ_{QCD}:**

$\sigma(s_T) \sim$



- **Factorization at twist-3 – initial-state:**



Normal twist-2 distributions

- **Twist-3 quark-gluon correlation:**

$$T_{q,F}(x,x,\mu_F) = \int \frac{dy_1^-}{2\pi} e^{ixP^+y_1^-} \langle P, s_T|\overline{\psi}_q(0)\frac{\gamma^+}{2}\left[\int dy_2^- \, \epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

SSA in QCD Collinear Factorization – II

Qiu, Sterman, 1998

❑ **Factorization formalism for SSA of single hadron:**

$$\Delta\sigma_{A+B\to\pi}(\vec{s}_T) = \sum_{abc} \phi^{(3)}_{a/A}(x_1, x_2, \vec{s}_T) \otimes \phi_{b/B}(x') \otimes H_{a+b\to c}(\vec{s}_T) \otimes D_{c\to\pi}(z)$$

$$+ \sum_{abc} \delta q^{(2)}_{a/A}(x, \vec{s}_T) \otimes \phi^{(3)}_{b/B}(x'_1, x'_2) \otimes H''_{a+b\to c}(\vec{s}_T) \otimes D_{c\to\pi}(z)$$

$$+ \sum_{abc} \delta q^{(2)}_{a/A}(x, \vec{s}_T) \otimes \phi_{b/B}(x') \otimes H'_{a+b\to c}(\vec{s}_T) \otimes D^{(3)}_{c\to\pi}(z_1, z_2)$$

$$+ \text{higher power corrections},$$

Only one twist-3 distribution in each term!

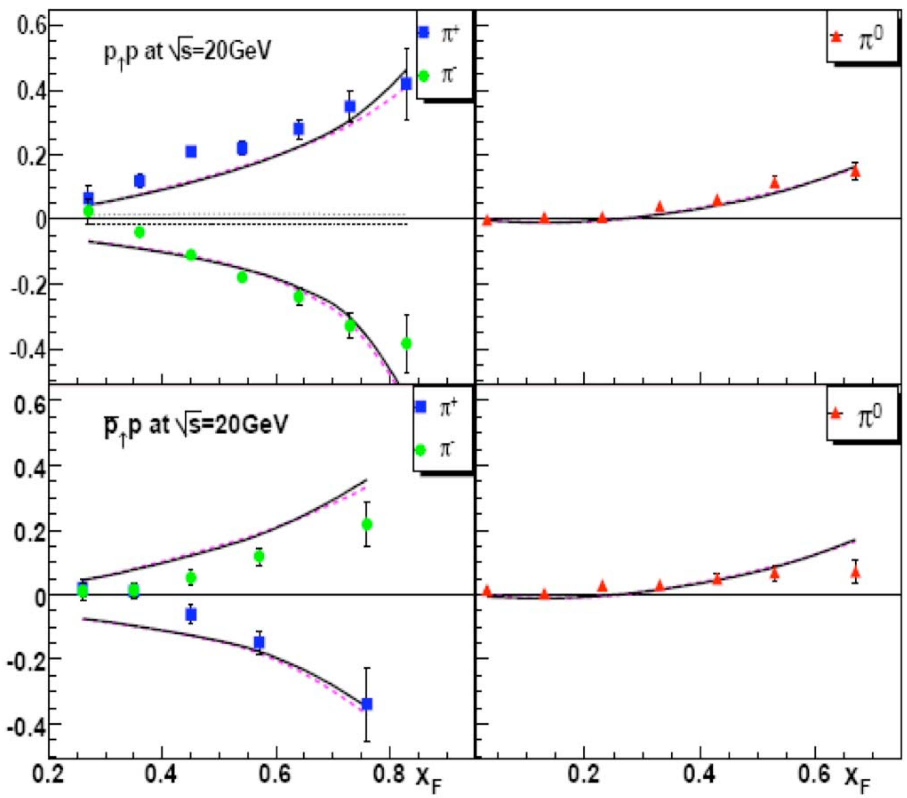
❖ 1st term: Collinear version of Sivers effect

❖ 2nd term: Collinear version of transversity + BM function

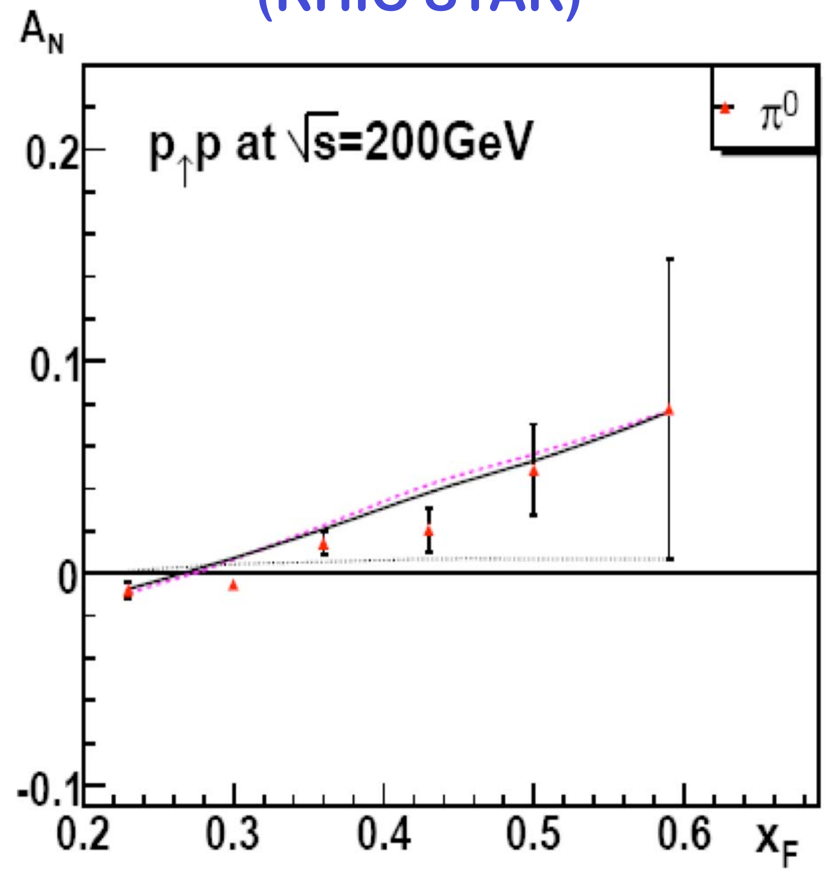
❖ 3rd term: Collinear version of Collins effect

Asymmetries from the $T_F(x,x)$

(FermiLab E704)



(RHIC STAR)



Kouvaris,Qiu,Vogelsang,Yuan, 2006

Nonvanish twist-3 function ⟹ Nonvanish transverse motion

Multi-gluon correlation functions

❑ Diagonal tri-gluon correlations:

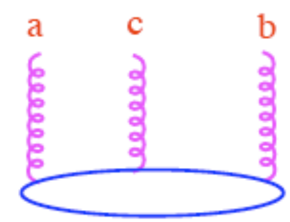
Ji, PLB289 (1992)

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+ y_1^-} \times \frac{1}{xP^+} \langle P, s_\perp | F^+{}_\alpha(0) \left[\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma{}^+(y_2^-) \right] F^{\alpha +}(y_1^-) | P, s_\perp \rangle$$

❑ Two tri-gluon correlation functions – color contraction:

$$T_G^{(f)}(x,x) \propto i f^{ABC} F^A F^C F^B = F^A F^C (\mathcal{T}^C)^{AB} F^B$$

$$T_G^{(d)}(x,x) \propto d^{ABC} F^A F^C F^B = F^A F^C (\mathcal{D}^C)^{AB} F^B$$



Quark-gluon correlation: $T_F(x,x) \propto \overline{\psi}_i F^C (T^C)_{ij} \psi_j$

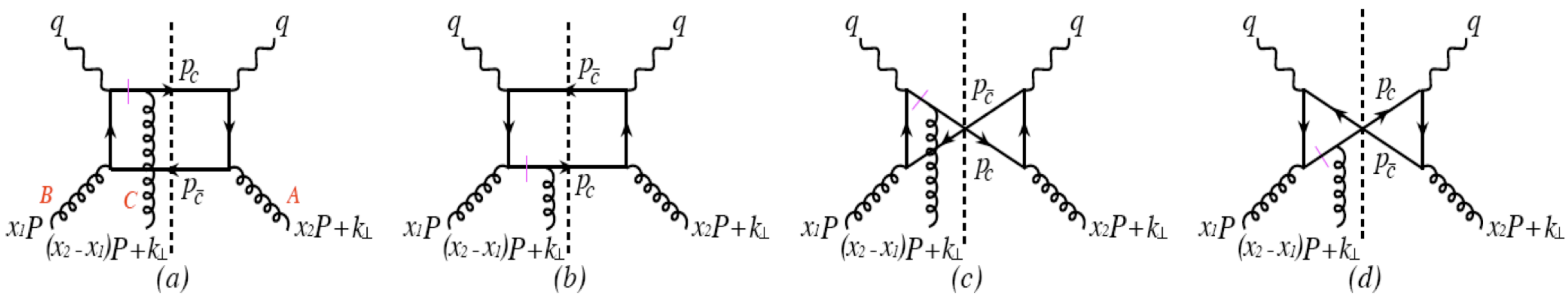
❑ D-meson production at EIC:

- ❖ Clean probe for gluonic twist-3 correlation functions
- ❖ $T_G^{(f)}(x,x)$ could be connected to the gluonic Sivers function

D-meson production at EIC

Kang, Qiu, PRD, 2008

❑ Dominated by the tri-gluon subprocess:



- ❖ Active parton momentum fraction cannot be too large
- ❖ Intrinsic charm contribution is not important
- ❖ Sufficient production rate

❑ Single transverse-spin asymmetry:

$$A_N = \frac{\sigma(s_\perp) - \sigma(-s_\perp)}{\sigma(s_\perp) + \sigma(-s_\perp)} = \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} \Bigg/ \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}$$

SSA is directly proportional to tri-gluon correlation functions

Features of the SSA in D-production at EIC

- Dependence on tri-gluon correlation functions:

$$D - \text{meson} \propto T_G^{(f)} + T_G^{(d)} \qquad \overline{D} - \text{meson} \propto T_G^{(f)} - T_G^{(d)}$$

Separate $T_G^{(f)}$ and $T_G^{(d)}$ by the difference between D and $\bar{D}$

- Model for tri-gluon correlation functions:

$$T_G^{(f,d)}(x,x) = \lambda_{f,d} G(x) \qquad \lambda_{f,d} = \pm\lambda_F = \pm 0.07\text{GeV}$$

- Kinematic constraints:

$$x_{min} = \begin{cases} x_B \left[1 + \frac{P_{h\perp}^2 + m_c^2}{z_h(1-z_h)Q^2}\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \geq 1 \\ x_B \left[1 + \frac{2m_c^2}{Q^2}\left(1 + \sqrt{1 + \frac{P_{h\perp}^2}{z_h^2 m_c^2}}\right)\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \leq 1 \end{cases}$$

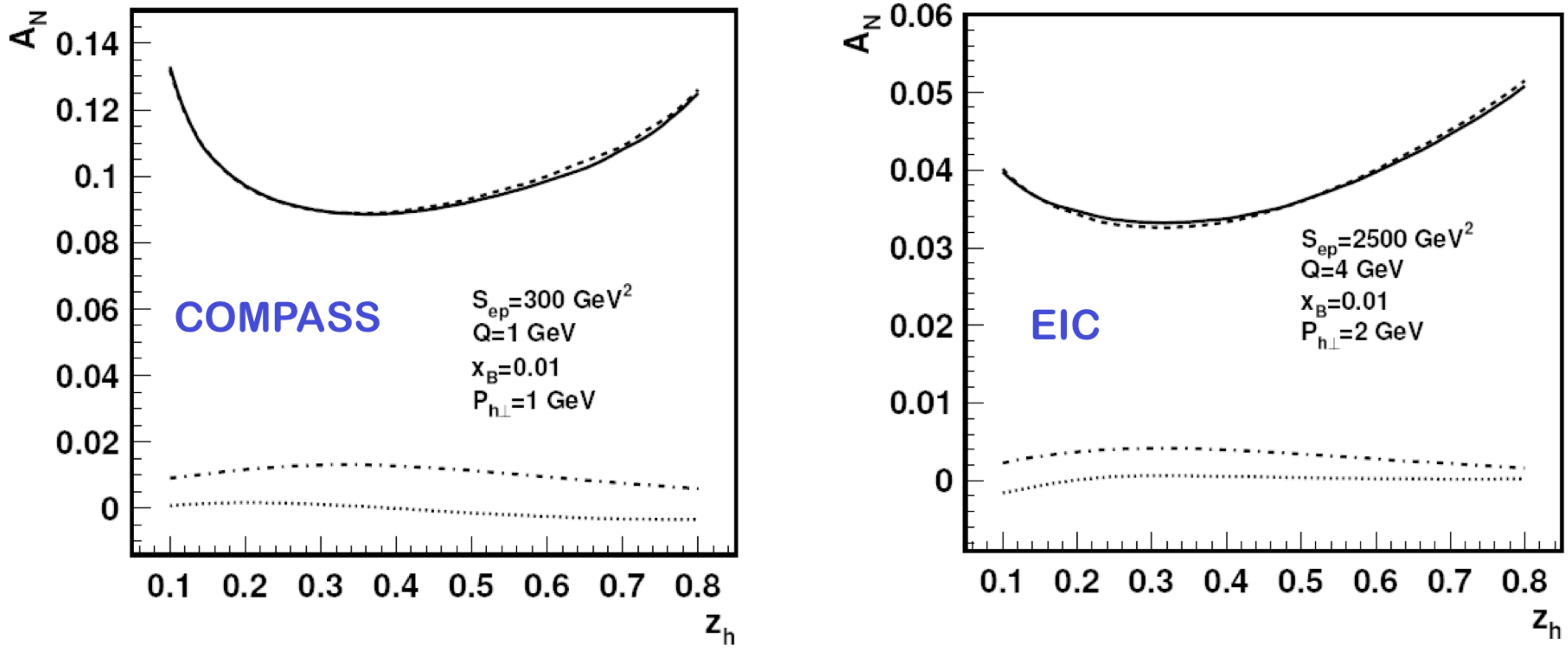
Note: The $z_h(1-z_h)$ has a maximum

SSA should have a minimum if the derivative term dominates

Minimum in the SSA of D-production at EIC

Kang, Qiu, PRD, 2008

- SSA for D^0 production (λ_f only):

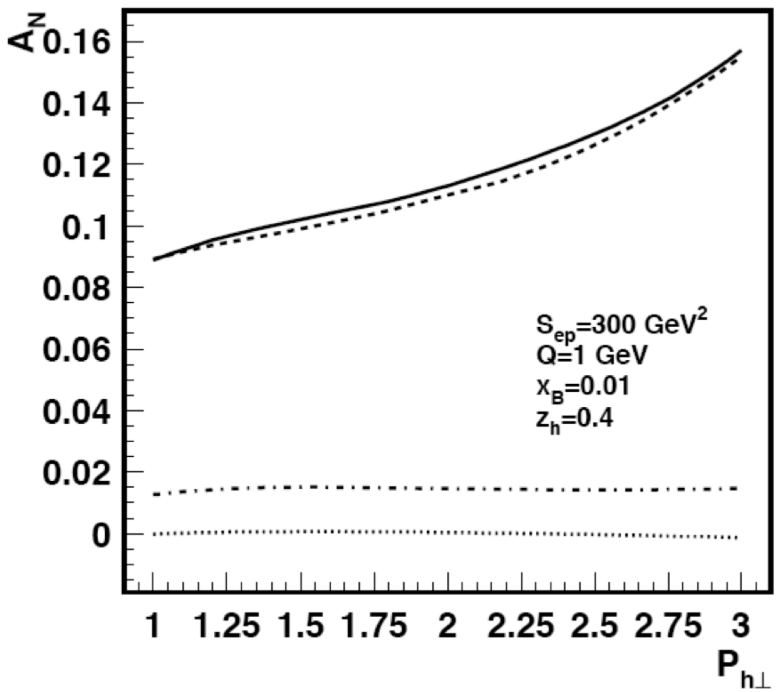


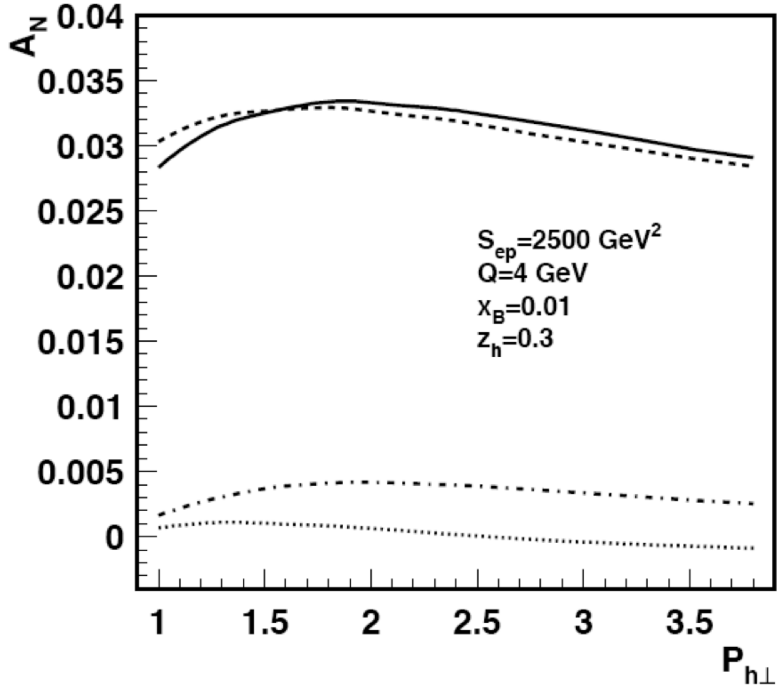
- Derivative term dominates, and small φ dependence
- Asymmetry is **twice** if $T_G^{(f)} = +T_G^{(d)}$, or **zero** if $T_G^{(f)} = -T_G^{(d)}$
- Opposite for the $\bar{D}$ meson
- Asymmetry has a minimum ~ z_h ~ 0.5

Maximum in the SSA of D-production at EIC

Kang, Qiu, PRD, 2008

- SSA for D^0 production (λ_f only):





- The SSA is a twist-3 effect, it should fall off as $1/P_T$ when $P_T >> m_c$
- For the region, $P_T \sim m_c$,

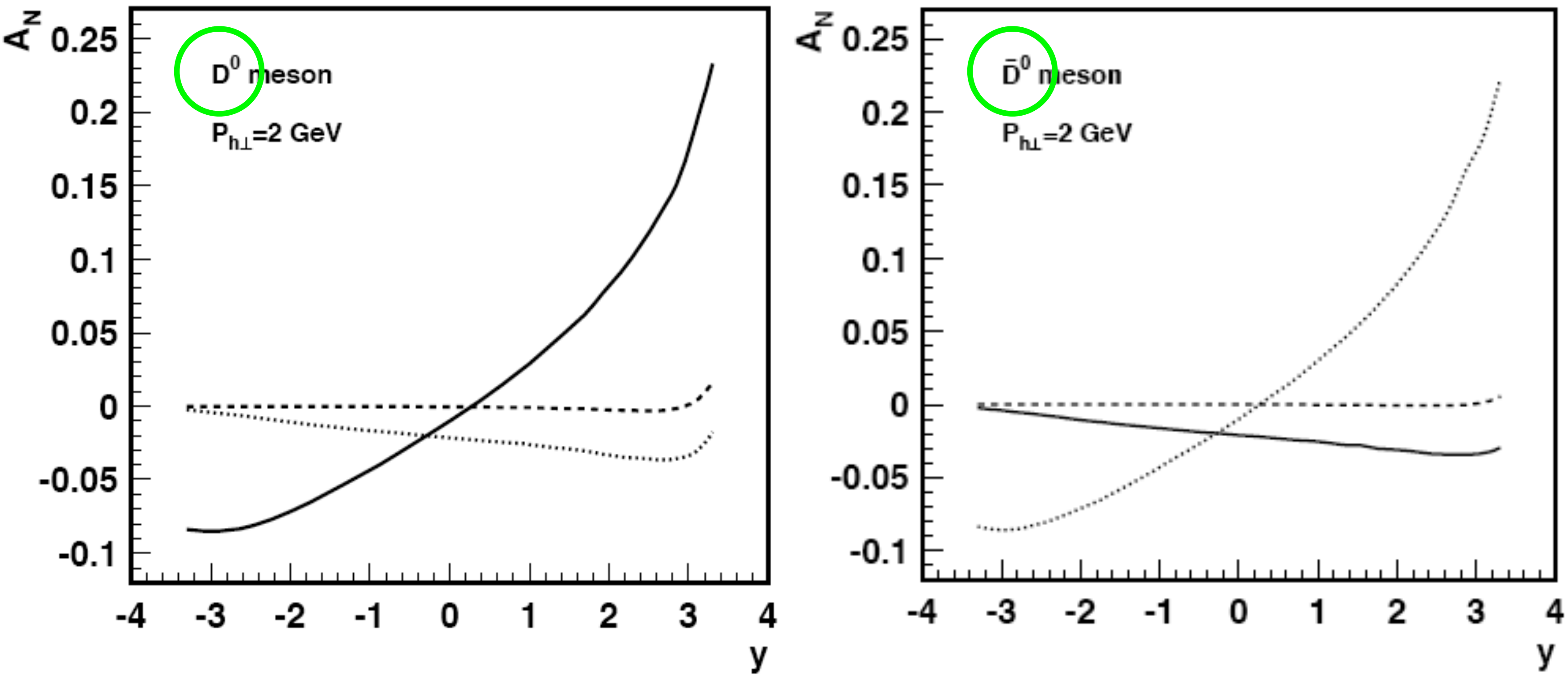
$$A_N \propto \epsilon^{P_h s_\perp n \bar{n}} \frac{1}{\tilde{t}} = -\sin\phi_s \frac{P_{h\perp}}{\tilde{t}}$$

$$\tilde{t} = (p_c - q)^2 - m_c^2 = -\frac{1-\hat{z}}{\hat{x}} Q^2$$

$$\hat{z} = z_h/z, \quad \hat{x} = x_B/x$$

SSA of D-meson production at RHIC

- **Rapidity:** $\sqrt{s} = 200\ \text{GeV} \quad \mu = \sqrt{m_c^2 + P_{h\perp}^2} \quad m_c = 1.3\ \text{GeV}$



Solid: $(1)\ \lambda_f = \lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = T_G^{(d)}$

Dashed: $(2)\ \lambda_f = \lambda_d = 0 \qquad T_G^{(f)} = T_G^{(d)} = 0$

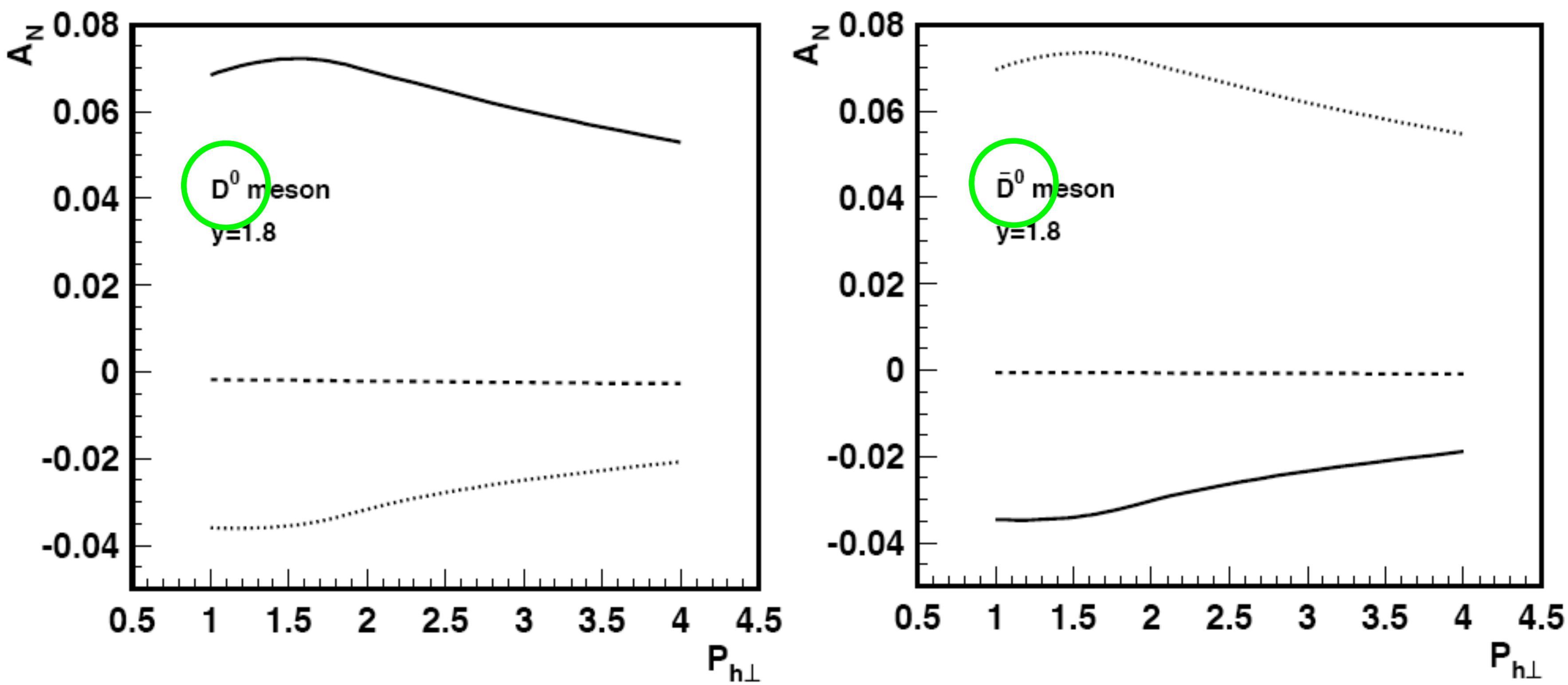
Dotted: $(3)\ \lambda_f = -\lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = -T_G^{(d)}$

No intrinsic Charm included

Kang, Qiu, Vogelsang, Yuan, 2008

SSA of D-meson production at RHIC

❑ **P_T dependence:** $\sqrt{s}\ =\ 200\ \text{GeV}\quad \mu=\sqrt{m_c^2+P_{h\perp}^2}\quad m_c=1.3\ \text{GeV}$



Solid: $(1)\ \lambda_f=\lambda_d=0.07\ \text{GeV}\qquad T_G^{(f)}=T_G^{(d)}$

Dashed: $(2)\ \lambda_f=\lambda_d=0\qquad T_G^{(f)}=T_G^{(d)}=0$

Dotted: $(3)\ \lambda_f=-\lambda_d=0.07\ \text{GeV}\qquad T_G^{(f)}=-T_G^{(d)}$

No intrinsic Charm included

Kang, Qiu, Vogelsang, Yuan, 2008

Scale dependence of SSA

- Almost all existing calculations of SSA are at LO:
  - Strong dependence on renormalization and factorization scales
  - Artifact of the lowest order calculation
- Improve QCD predictions:
  - Complete set of twist-3 correlation functions relevant to SSA
  - LO evolution for the universal twist-3 correlation functions
  - NLO partonic hard parts for various observables
  - NLO evolution for the correlation functions, …
- Current status:
  - Two sets of twist-3 correlation functions
  - LO evolution kernel for $T_{q,F}(x,x)$ and $T_{G,F}^{(f,d)}(x,x)$ Kang, Qiu, 2009
  - NLO hard part for SSA of p_T weighted Drell-Yan Vogelsang, Yuan, 2009

Two sets of twist-3 correlation functions

❑ Twist-2 distributions:

❖ Unpolarized PDFs:

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

❖ Polarized PDFs:

$$\Delta q(x) \propto \langle P,S_{\|}|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P,S_{\|}\rangle$$
$$\Delta G(x) \propto \langle P,S_{\|}|F^{+\mu}(0)F^{+\nu}(y)|P,S_{\|}\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 correlation functions:

Kang, Qiu, PRD, 2009

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Evolution equations and evolution kernels

- Evolution equation is a consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q)H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)}\right) \otimes f_3$

- Evolution kernel is process independent:

  - Calculate directly from the variation of process independent twist-3 distributions

    Kang, Qiu, 2009
    Yuan, Zhou, 2009

  - Extract from the scale dependence of the NLO hard part of any physical process

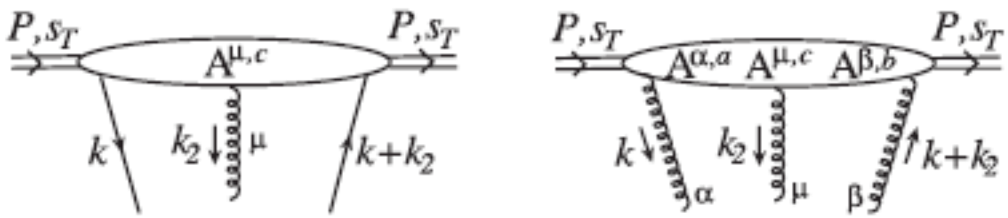
    Vogelsang, Yuan, 2009

  - Both approaches should give the same kernel

Evolution equations – I

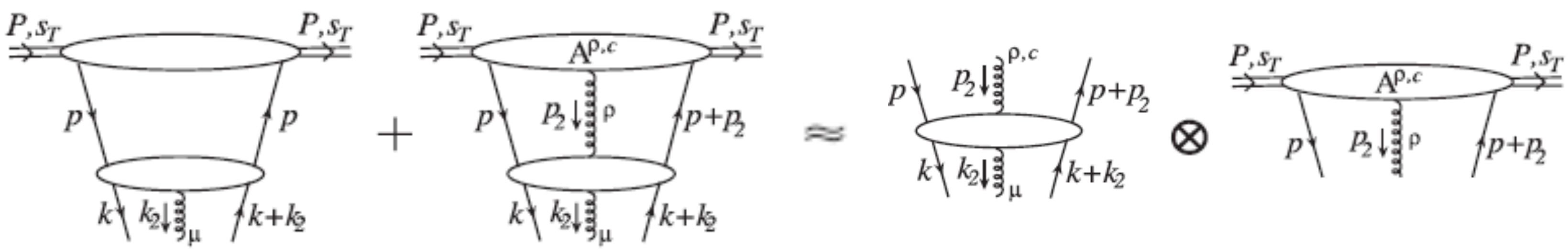
- Feynman diagram representation of twist-3 distributions:

Kang, Qiu, 2009



Different twist-3 distributions ⇔ diagrams with different cut vertices

- Collinear factorization of twist-3 distributions:



- Cut vertex and projection operator in LC gauge:

$$\mathcal{V}^{\,\rm LC}_{q,F} = \frac{\gamma^+}{2P^+}\,\delta\!\left(x - \frac{k^+}{P^+}\right) x_2\,\delta\!\left(x_2 - \frac{k_2^+}{P^+}\right)\left(i\epsilon^{s_T\sigma n\bar{n}}\right)\left[-g_{\sigma\mu}\right]\mathcal{C}_q$$

$$\mathcal{P}^{\,\rm (LC)}_{q,F} = \frac{1}{2}\,\gamma\cdot P\left(\frac{-1}{\xi_2}\right)\left(i\epsilon^{s_T\rho n\bar{n}}\right)\tilde{\mathcal{C}}_q$$

Evolution equations – II

- Closed set of evolution equations (spin-dependent):

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T) = \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{qq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K_{q\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$
$$+ \sum_{i=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(i)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{qg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{q\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)].$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x, x+x_2, \mu_F, s_T) = \sum_{j=f,d} \int d\xi d\xi_2 [\tilde{\mathcal{T}}^{(j)}_{G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{gg}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}^{(j)}_{\Delta G,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(ji)}_{g\Delta g}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)]$$
$$+ \sum_q \int d\xi d\xi_2 [\tilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{gq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$
$$+ \tilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) K^{(i)}_{g\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)],$$

Plus two more equations for:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x, x+x_2, \mu_F, s_T) \quad \text{and} \quad \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F, s_T)$$

Evolution equations – III

❑ Distributions relevant to SSA:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}_{q,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T) + \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{q,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}^{(i)}_{G,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x, x+x_2, \mu_F, s_T) + \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{G,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}_{\Delta q,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x, x+x_2, \mu_F, s_T) - \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}_{\Delta q,F}(x+x_2, x, \mu_F, s_T)\right],$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \mathcal{T}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F) = \frac{1}{2}\left[\mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x, x+x_2, \mu_F, s_T) - \mu_F^2 \frac{\partial}{\partial \mu_F^2} \tilde{\mathcal{T}}^{(i)}_{\Delta G,F}(x+x_2, x, \mu_F, s_T)\right].$$

❑ Important symmetry property:

$$T_{\Delta q,F}(x, x, \mu_F) \equiv \int dx_2 [2\pi\delta(x_2)] \mathcal{T}_{\Delta q,F}(x, x+x_2, \mu_F) = 0,$$

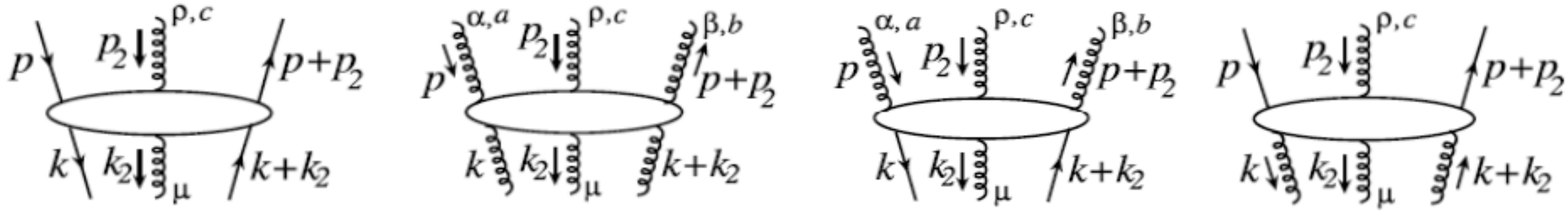
$$T^{(f,d)}_{\Delta G,F}(x, x, \mu_F) \equiv \int dx_2 [2\pi\delta(x_2)] \left(\frac{1}{x}\right) \mathcal{T}^{(f,d)}_{\Delta G}(x, x+x_2, \mu_F) = 0.$$

These two correlation functions do not give the gluonic pole contribution directly

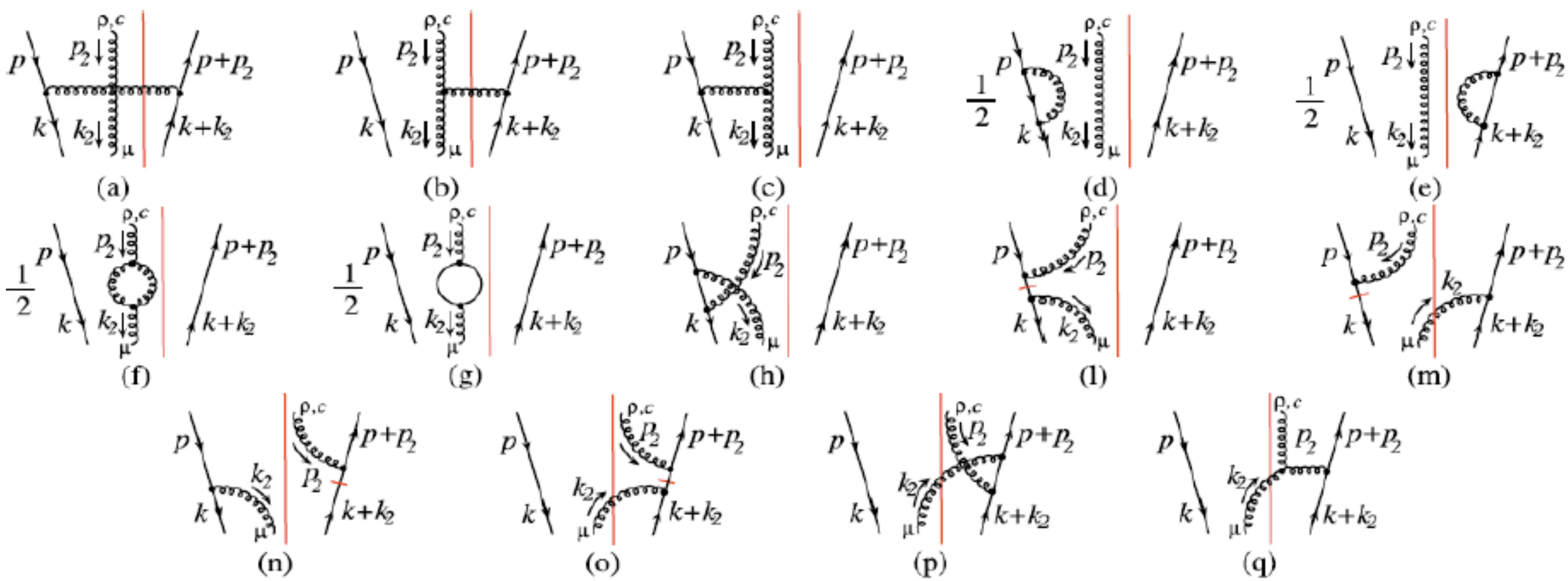
Evolution kernels

Kang, Qiu, PRD, 2009

❑ Feynman diagrams:



❑ LO for flavor non-singlet channel:



Leading order evolution equations – I

Kang, Qiu, PRD, 2009

❑ Quark:

$$\frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{\boxed{P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu_F)}\right.$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu_F)-T_{q,F}(\xi,\xi,\mu_F)\right]+z\,T_{q,F}(\xi,x,\mu_F)\right]+\frac{C_A}{2}\left[T_{\Delta q,F}(x,\xi,\mu_F)\right]$$
$$\left.+P_{qg}(z)\left(\frac{1}{2}\right)\boxed{\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)+T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]}\right\}$$

❑ Antiquark:

$$\frac{\partial T_{\bar{q},F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{\boxed{P_{qq}(z)\,T_{\bar{q},F}(\xi,\xi,\mu_F)}\right.$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{\bar{q},F}(\xi,x,\mu_F)-T_{\bar{q},F}(\xi,\xi,\mu_F)\right]+z\,T_{\bar{q},F}(\xi,x,\mu_F)\right]+\frac{C_A}{2}\left[T_{\Delta \bar{q},F}(x,\xi,\mu_F)\right]$$
$$\left.+P_{qg}(z)\left(\frac{1}{2}\right)\boxed{\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)-T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]}\right\}$$

- ❖ All kernels are infrared safe
- ❖ Diagonal contribution is the same as that of DGLAP
- ❖ Quark and antiquark evolve differently – caused by tri-gluon

Leading order evolution equations – II

Kang, Qiu, PRD, 2009

❑ Gluons:

$$\frac{\partial T^{(d)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{\boxed{P_{gg}(z)\, T^{(d)}_{G,F}(\xi,\xi,\mu_F)}\right.$$
$$+\frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(d)}_{G,F}(\xi,x,\mu_F)-T^{(d)}_{G,F}(\xi,\xi,\mu_F)\right]\right.$$
$$\left.+2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(d)}_{G,F}(\xi,x,\mu_F)+(1+z)\,T^{(d)}_{\Delta G,F}(x,\xi,\mu_F)\right]$$
$$\left.+P_{gq}(z)\left(\frac{N_c^2-4}{N_c^2-1}\right)\boxed{\sum_q\left[T_{q,F}(\xi,\xi,\mu_F)+T_{\bar q,F}(\xi,\xi,\mu_F)\right]}\right\}$$

Similar expression for $T^{(f)}_{G,F}(x,x,\mu_F)$

- ❖ Kernels are also infrared safe
- ❖ diagonal contribution is the same as that of DGLAP
- ❖ Two tri-gluon distributions evolve slightly different
- ❖ $T^{(d)}_{G,F}$ has no connection to TMD distribution
- ❖ Evolution can generate $T^{(d)}_{G,F}$ as long as $\sum_q [T_{q,F}+T_{\bar q,F}] \neq 0$

Leading order evolution equations – III

- Evolution equations for diagonal correlation functions are not closed!

- Model for the off-diagonal correlation functions:

For the symmetric correlation functions:

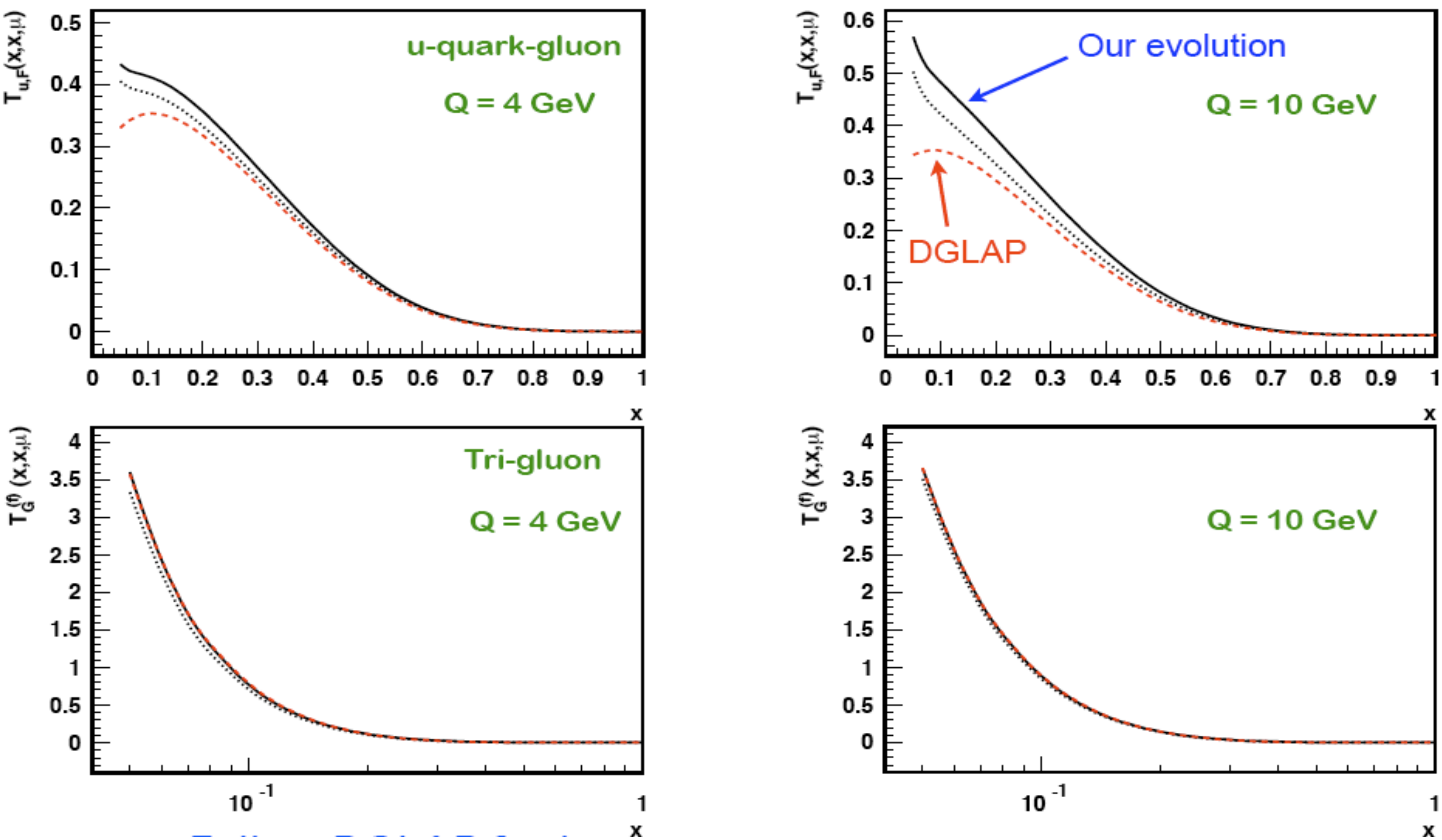
$$T_{q,F}(x_1, x_2, \mu_F) = \tfrac{1}{2}[T_{q,F}(x_1, x_1, \mu_F) + T_{q,F}(x_2, x_2, \mu_F)]e^{-[(x_1-x_2)^2/2\sigma^2]},$$

$$\mathcal{T}^{(f,d)}_{G,F}(x_1, x_2, \mu_F) = \tfrac{1}{2}[\mathcal{T}^{(f,d)}_{G,F}(x_1, x_1, \mu_F) + \mathcal{T}^{(f,d)}_{G,F}(x_2, x_2, \mu_F)]e^{-[(x_1-x_2)^2/2\sigma^2]},$$

⟹

$$T^{(f,d)}_{G,F}(x_1, x_2, \mu_F) = \frac{1}{2}\left[T^{(f,d)}_{G,F}(x_1, x_1, \mu_F) + \frac{x_2}{x_1}T^{(f,d)}_{G,F}(x_2, x_2, \mu_F)\right]e^{-[(x_1-x_2)^2/2\sigma^2]}.$$

Scale dependence of twist-3 correlations



- ❖ Follow DGLAP at large x
- ❖ Large deviation at low x (stronger correlation)

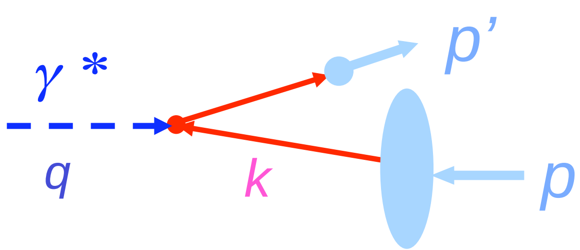
Kang, Qiu, PRD, 2009

TMD factorization

- Need processes with two observed momentum scales:

$$Q_1 \gg Q_2 \quad \begin{cases} Q_1 & \text{necessary for pQCD factorization to have a chance} \\ Q_2 & \text{sensitive to parton's transverse motion} \end{cases}$$

- Example – semi-inclusive DIS:



  - Both p and p' are observed
  - p'_T probes the parton's k_T
  - Effect of k_T is not suppressed by Q

- Very limited processes with valid TMD factorization

  - Drell-Yan transverse momentum distribution: Q, q_T
    - o quark Sivers function
    - o low rate

Collins, Qiu, 2007
Vogelsang, Yuan, 2007

  - Semi-inclusive DIS for light hadrons: Q, p_T
    - o mixture of quark Sivers and Collins function

TMD parton distributions

- SIDIS:

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2\mathbf{y}_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp}\, \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\Phi_n^\dagger(\{\infty,0\},\mathbf{0}_\perp)$$
$$\times\, \Phi_{\mathbf{n}_\perp}^\dagger(\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\frac{\gamma^+}{2}\Phi_n(\{\infty,y^-\},\mathbf{y}_\perp)\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

Gauge links:

$$\Phi_n(\{\infty,y^-\},\mathbf{y}_\perp) \equiv \mathcal{P}e^{-ig\int_{y^-}^{\infty} dy_1^- n^\mu A_\mu(y_1^-,\mathbf{y}_\perp)}$$
$$\Phi_{\mathbf{n}_\perp}(\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\}) \equiv \mathcal{P}e^{-ig\int_{\mathbf{0}_\perp}^{\mathbf{y}_\perp} d\mathbf{y}'_\perp \mathbf{n}_\perp^\mu A_\mu(\infty,\mathbf{y}'_\perp)}$$

- Drell-Yan:

$$f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2\mathbf{y}_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp}\langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\Phi_n^\dagger(\{-\infty,0\},\mathbf{0}_\perp)$$
$$\times\, \Phi_{\mathbf{n}_\perp}^\dagger(-\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\frac{\gamma^+}{2}\,\Phi_n(\{-\infty,y^-\},\mathbf{y}_\perp)\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

- PT invariance:

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

Collins 2002
Boer et al, 2003
Kang, Qiu, 2009

- Sivers function:

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)\,\vec{S}\cdot(\hat{p}\times\hat{\mathbf{k}}_\perp)$$

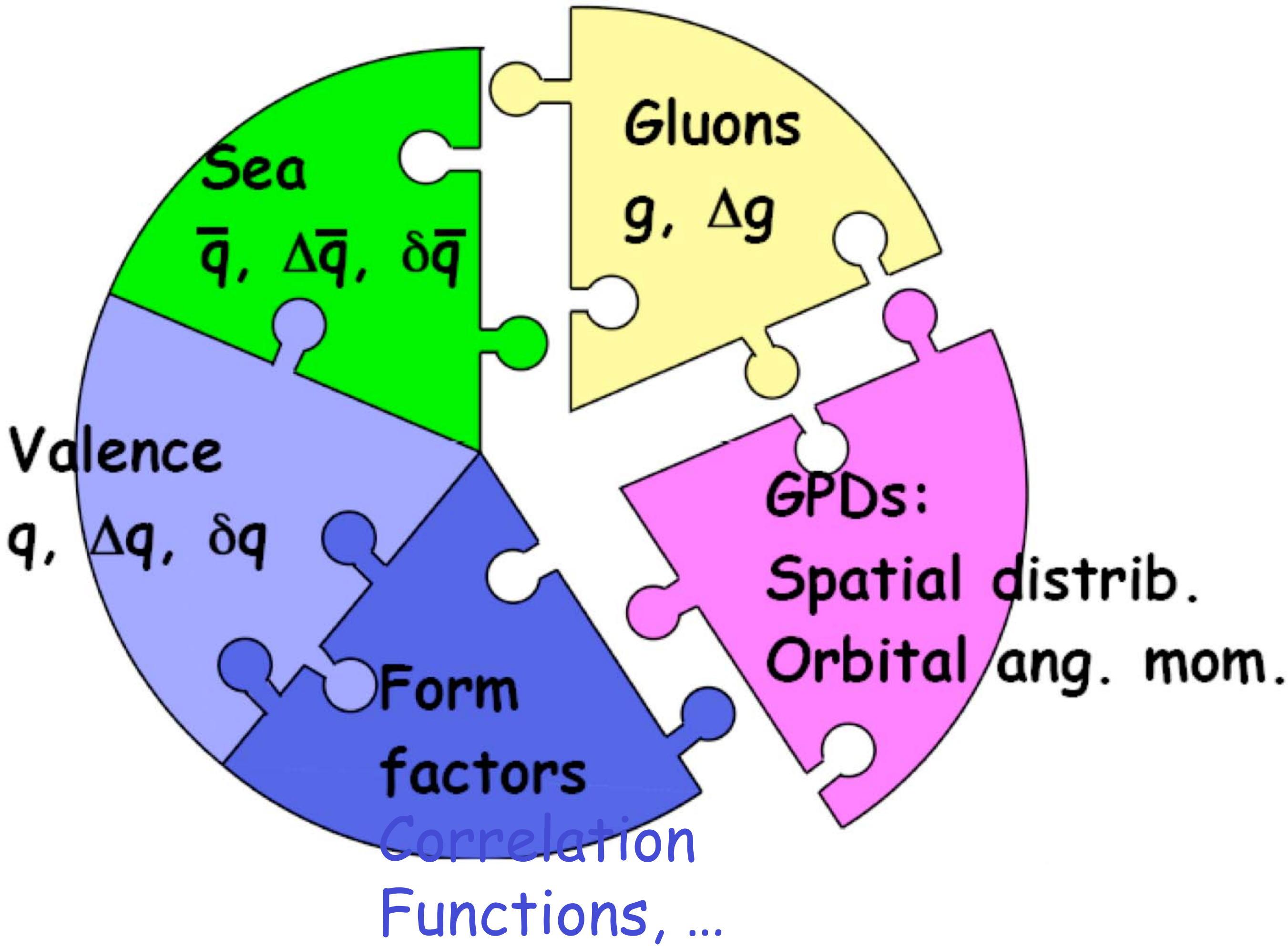
$\Longrightarrow$ $$f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)^{\rm SIDIS} = -f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)^{\rm DY}$$ $\Longleftarrow$ **Modified Universality**

Summary and outlook

- It seems likely that quark and gluon helicity alone is not sufficient to make up the proton's spin
- SSA is connected to parton's transverse motion
- Collinear factorization and the TMD factorization cover different kinematic regimes – they are consistent when they are overlap
- Twist-3 factorization formalism seems to be on a solid ground
- Spin program opens a whole new meaning to test QCD dynamics!

Thank you!

Challenge: Map out the nucleon

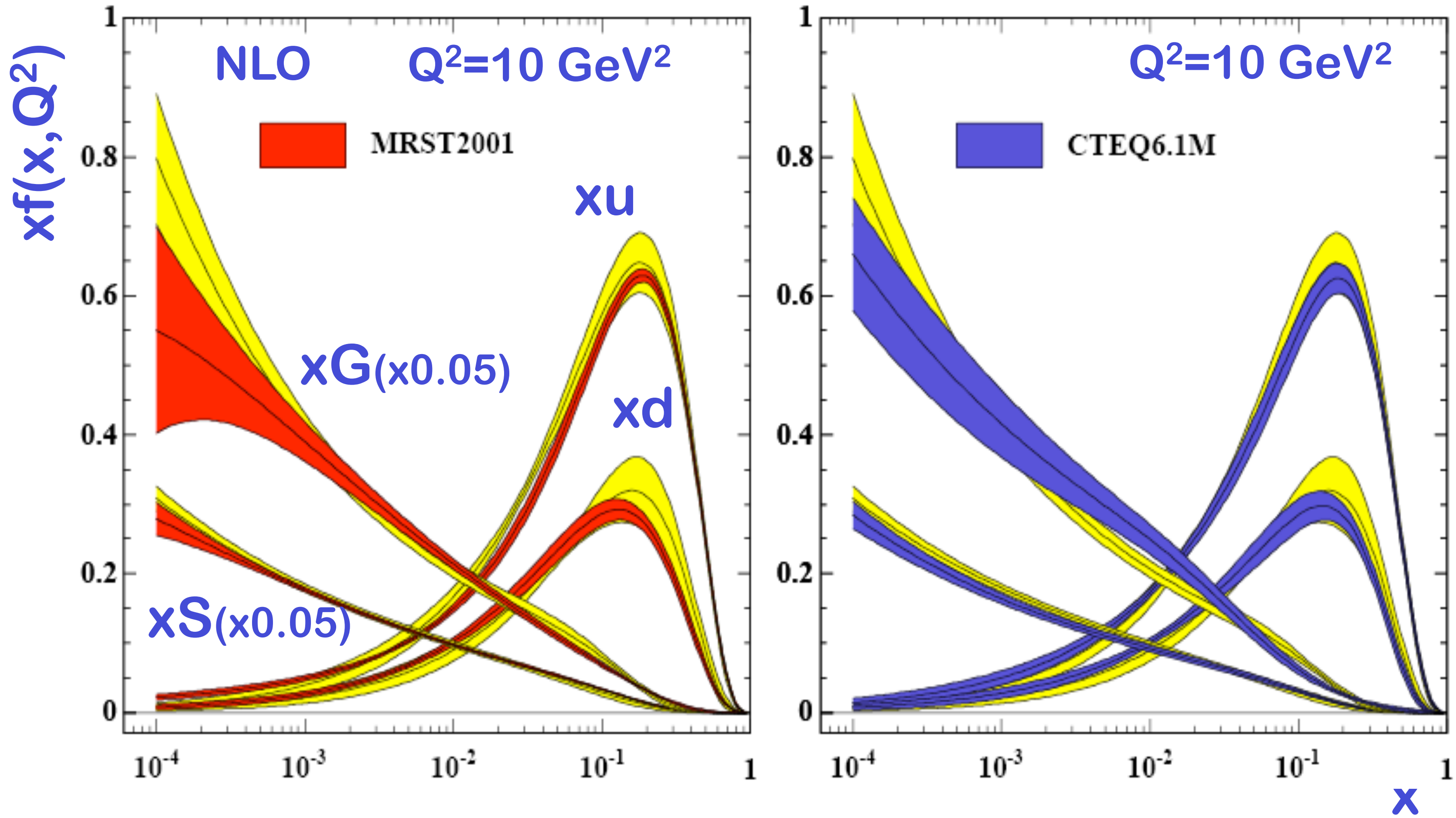


RHIC spin and future EIC spin will play a key role!

Backup transparencies

Universal parton distributions

❑ Modern sets of PDFs with uncertainties:



Consistently fit almost all unpolarized data with Q > 2GeV